Unified model for Vehicle Routing Problem with Synchronization Requirements

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Abstract. This technical report presents mathematical model for Vehicle Routing Problem with Synchronization Requirements according to its spatial and temporal dimension. The spatial dimension of synchronization defines the locations where vehicle synchronization can take place. In Types I and II, the vehicles have to meet at fixed and given synchronization points, whereas in Types III and IV, synchronization points are an outcome of the planning. The temporal dimension defines the order in which vehicles must visit a synchronization point. In Types I and III, synchronization points must be served simultaneously by vehicles, whereas in Types II and IV, a precedence relation must be respected for the vehicles. For supporting the planning of logistics service operations, we formulate a unified mixed-integer programming model for vehicle routing, where vehicle routes are synchronized at fixed and given points.

1 Introduction

Vehicle Routing Problem with Synchronization Requirements is one of the currently most investigated extensions to the traditional Vehicle Routing Problem. Synchronization means that two or more vehicles or service operators must visit a location. It means that the routes depend on each other according to their spatial and temporal dimension and, thus, they must be planned together. The following time-space classification of synchronization requirements is proposed in Mankowska et al. (2011). In the temporal dimension of synchronization, simultaneous synchronization and synchronization with given precedence are distinguished. The spatial aspect implies whether the location at which the vehicles meet is known before the planning or if it is an outcome of the planning. By combining temporal and spatial synchronization aspects, four types of synchronization are considered, see Table 1.
Temporal synchronization simultaneous with precedence

<table>
<thead>
<tr>
<th>Spatial synchronization</th>
<th>fixed points</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable points</td>
<td>Type III</td>
<td>Type IV</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Synchronization types.

2 Mathematical formulation

In the following, a unified mathematical programming model for Vehicle Routing Problem with Synchronization Requirements is presented. The used notation is presented in Table 2.

(1) comprises conceivable objectives. Here, $D$ corresponds to the total distance traveled by the vehicles, $WT$ measures to the total waiting time of the vehicles, $T_{\text{max}}$ describes the latest arrival at the depot of the latest vehicle, $L$ indicates the largest lateness of a visit by a customer and $F$ corresponds to the fairness goal, which can be measured by the maximal lateness of all visits.

$$ZF = \{D, WT, T_{\text{max}}, L, F, \ldots \}$$

Constraints (2) allow the routes of the vehicles start and end at the depot.

$$\sum_{s \in D} \sum_{j \in C_0} x_{0js} = \sum_{s \in D} \sum_{j \in C_0} x_{0vs} = 1 \quad \forall v \in V$$

If customer $i$ requires service of type $s$, exactly one vehicle has to move to this customer, see constraints (3).

$$\sum_{v \in V} \sum_{j \in C_0} x_{ijvs} = 1 \quad \forall i \in C, \sum_{s \in D} r_{is} = 1$$

Constraints (4) control the flow of the routing variable.

$$\sum_{s \in D} \sum_{j \in C_0} x_{jvs} = \sum_{s \in D} \sum_{j \in C_0} x_{ijus} \quad \forall i \in C_0, v \in V$$

Constraints (5) ensure the time flow of the variables, i.e. if vehicle $v$ moves from node $i$ to node $j$ to provide there service type $s_2$, it has to start its service at the subsequent node $j$ by at least processing time and the travel time later than at node $i$.

$$t_{ivs_1} + p_{ivs_1} + d_{ij} \leq t_{jvs_2} + M_1 \cdot (1 - x_{ijvs_2}) \quad \forall i, j \in C_0, v \in V, s_1, s_2 \in D$$
notation

\[ C \] set of customers
\[ S \] set of potential synchronization points
\[ C^0 \] set of all nodes in the network including depot (node 0)
\[ V \] set of all vehicles
\[ D \] set of service types
\[ r_{is} \] service requirement of customer \( i \) for service type \( s \)
\[ p_{ivs} \] service duration for serving customer \( i \) by vehicle \( v \) for service type \( s \)
\[ d_{ij} \] travel time between nodes \( i \) and \( j \)
\[ e_i, l_i \] time window of customer \( i \)
\[ q_i \] demand of customer \( i \)
\[ a_{ivs} = \{0, 1\} \] accessibility of customers \( i \) by vehicle \( v \) for providing service type \( s \)
\[ \beta = \{a_{ivs} \cdot r_{is}, 2\} \] synchronization requirement of the customers: Type I and Type II: according to the accessibility \( a_{ivs} \) (here, this parameter can be considered as qualifications of a staff member driving a vehicle) and the service requirements \( r_{is} \)
\[ \delta_{i}^{min}, \delta_{i}^{max} \] minimal and maximal time gap between the synchronized visits at node \( i \)

decision variables

\[ x_{ijvs} \] binary, take value 1 iff vehicle \( v \) moves directly from node \( i \) to node \( j \) to provide there service type \( s \)
\[ \alpha_{iv} \] binary, takes value 1 if vehicle \( v \) plays a receiving role at synchronization node \( i \)
\[ u_i \] binary, takes value 1 if synchronization node \( i \) is used in the solution

- service start time if \( i \in C \)
- start time of loading or unloading operation if \( i \in S \)
- time of return if \( i = 0 \)
\[ z_{iv} \] lateness of vehicle \( v \) at customer \( i \)
\[ Q_{iv} \] load of vehicle \( v \) upon arrival at node \( i \)

Table 2. Notation.

Constraints (6) ensures that vehicles arrive at a customer after the start of time window \( e_i \) and measure the lateness \( z_{iv} \) if a service type \( s \) starts after the end of the corresponding time window of customer \( i \), namely \( l_i \).

\[ e_i \leq t_{ivs} \leq l_i + z_{iv} \quad \forall i \in C : \sum_{j \in C^0} x_{ijv}=1, \quad v \in V, s \in D \quad (6) \]
The resulting load reduction following from serving the customers is expressed by constraints (7).

\[ Q_{iv} - q_i \geq Q_{jv} - M_2 \cdot (1 - \sum_{s \in D} x_{ijvs}) \quad \forall i \in C, j \in C^0, v \in V \]  

(7)

If a synchronization point \( i \) is used within the planning because (a) the customer \( i \) requires synchronized service \( (\beta = a_{ivs} \cdot r_{is}) \) or (b) cargo transfer takes place at node \( i \) \( (\beta = 2) \), then two vehicles have to move to this node.

\[ \sum_{v \in V} \sum_{j \in C^0} x_{jivs} = \beta \cdot u_i \quad \forall i \in S, s \in D \]  

(8)

The role (supplying or receiving) of each vehicle is ensured by (9). Here, if current load of vehicle \( v \) upon arrival at a synchronization node \( i \) exceed the demand of customers that this vehicle visits, \( \alpha_{iv} \) is set to 1 to make this inequality feasible and, thus, this vehicle plays a receiving role at this node.

\[ \sum_{j \in C^0 \setminus S} q_j \sum_{s \in D} x_{jivs} \leq Q_{iv} + M_3 \cdot \alpha_{iv} \quad \forall i \in S, v \in V \]  

(9)

Constraints (10) ensure that a synchronization node is used only if one of the vehicles plays a receiving role.

\[ \sum_{v \in V} \alpha_{iv} = u_i \quad \forall i \in S \]  

(10)

Constraints (11) ensures that the role can be assigned to the vehicles if they move to a synchronization node.

\[ \alpha_{iv} \leq \sum_{j \in C^0} \sum_{s \in D} x_{jivs} \quad \forall i \in S, v \in V \]  

(11)

The minimal and the maximal time gap between the vehicles when visiting the synchronization nodes is ensured by (12) and (13), respectively.

\[ t_{iv2s2} - t_{iv1s1} \geq \delta^\text{min}_i - M_4 (4 - \alpha_{iv2} - u_i - \sum_{j \in C^0} x_{ijv1s1} - \sum_{k \in C^0} x_{ikv2s2}) \quad \forall i \in S, s_1 < s_2 \in D, v_1, v_2 \in V : v_1 \neq v_2 \]  

(12)

\[ t_{iv2s2} - t_{iv1s1} \leq \delta^\text{max}_i + M_5 (4 - \alpha_{iv2} - u_i - \sum_{j \in C^0} x_{ijv1s1} - \sum_{k \in C^0} x_{ikv2s2}) \quad \forall i \in S, s_1 < s_2 \in D, v_1, v_2 \in V : v_1 \neq v_2 \]  

(13)
Cargo transfer is ensured by (14).

\[ Q_{iv_1} + Q_{iv_2} \geq Q_{jv_1} + Q_{kv_2} - M_6 (3 - u_i - \sum_{s \in \mathcal{D}} x_{ijv_1s} - \sum_{s \in \mathcal{D}} x_{ikv_2s}) \]
\[ \forall i \in \mathcal{S}, j, k \in \mathcal{C}^0, v_1, v_2 \in \mathcal{V} : v_1 \neq v_2 \]  

(14)

The domains of the decision variables are determined by (15) - (17).

\[ x_{ijv_1s}, \alpha_{iv_1}, u_i \in \{0, 1\} \]  
\[ t_{iv_1}, z_{iv_1}, Q_{iv_1} \in \mathbb{R}_+ \]  
\[ \beta = \{a_{iv_1s} \cdot r_{is}, 2\} \]  

(15) (16) (17)

### 3 How to use the model?

This model can be used for all synchronization types presented in [1]. Because the real-world applications of synchronization of Types I and II are mostly service driven and synchronization of Types III and IV are mostly cargo delivery driven, further refinements can be made to make the proposed model tighter.

For synchronization of Types I and II, constraints (7), (9), (10), (11), (14) can be neglected and variable \( \beta \) is set to \( a_{iv_1s} \cdot r_{is} \). In addition, in constraints (8), variable \( u_i \) is fixed and take value 1.

For synchronization of Types III and IV, all constraints are relevant but the index \( s \) can be neglected if cargo delivery is the only service taken into consideration.

### 4 Conclusions

This technical report proposes a mathematical formulation for all synchronization types proposed in [Mankowska et al. (2011)] and also the refinements regarding the particular types.

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Bibliography