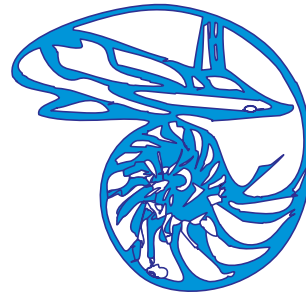


**EUROPEAN
MECHANICS
SOCIETY**



EUROMECH Colloquium 527

SHELL-LIKE STRUCTURES



NONCLASSICAL THEORIES AND APPLICATIONS

August 22nd–26th, 2011

Leucorea, Lutherstadt Wittenberg, Germany

Book of abstracts

Holm Altenbach & Victor Eremeyev (Editors)

Halle (Saale), Germany, 2011

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Preface

Shell-like structures are used in Civil and Aero-space engineering as basic elements of constructions. Such structures are used as a model of analysis in other branches, e.g. Mechanical engineering, but also in new branches like Medicine. New applications are primarily related to new materials - instead of steel or concrete, now one has to analyze sandwiches, laminates, foams, nano-films, biological membranes, etc. The new trends in applications demand improvements of the theoretical foundations of shell theory, since new effects must be taken into account. For example, in the case of small-size shell-like structures (nano-tubes) the surface effect plays an important role in the mechanical analysis of these structural elements.

The colloquium should be a forum of international experts discussing the new tendencies. It is addressed to scientists and researchers from industries. The focus is related to the following problems: new theories (based on two-dimensional field equations but describing non-classical effects), new constitutive equations (for materials like sandwiches, foams, etc. and which can be combined with the two-dimensional shell equations), complex structures (folded, branching and/or self-intersecting shell structures, etc.) and shell-like structures on different scales (for example: nano-tubes) or very thin structures (similar to membranes, but having a compression stiffness). In addition, phase transitions in shells and refined shell thermodynamics will be discussed.

This book of abstracts contains the abstracts submitted to the EUROMECH 527 Colloquium **Shell-like structures - Non-classical Theories and Applications**, which will be organized at the *Leucorea* (Lutherstadt Wittenberg, Germany) from August 22nd up to 26th, 2011.

Halle (Saale),
October, 2010

Holm Altenbach
Victor A. Eremeyev

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Non-classical Spatial Boundary Value Problems of Statics and Dynamics of Shells and Asymptotic Method of Their Solution

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Abstract Classical and improved theories of shells consider only one class of boundary value problems - on the facial surfaces of the shell the values of the corresponding stresses tensor components are given. From the position of supplements the problems, when on the facial surfaces other conditions - displacement vector, mixed boundary conditions of elasticity theory are given, are also very important (non-classical boundary value problems of theory of shells).

It is proved, that Kirchhoff-Love hypotheses of classical theory of shells are not applicable for the solution of these class of problems. It is shown, that they can be solved by asymptotic method of solution of singularly perturbed differential equations.

Noncontradictory asymptotic orders of stresses tensor components and displacement vector are established. Iteration processes for the determination of all the sought values with the beforehand given asymptotic exactness are built. As private cases, the solutions of static and dynamic problems, illustrating effectiveness of the asymptotic method, are brought. Non-classical special problems on free and forced vibrations of isotropic and anisotropic shells are considered. Frequencies and forms of free vibrations, the amplitudes of forced vibrations are determined. The effectiveness of the method for the solution of layered shells problems is shown.

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Analytical Solution of the Bending of Plates on a Functionally-graded Layer of a Complicated Structure

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Abstract The problem of interaction between thin plate and a supporting elastic foundation is reduced to that of solving system of an dual integral equations for the unknown normal contact pressure and differential equation of plate bend . The supporting medium is an isotropic elastic functionally-graded layer of a complicated structure of constant thickness lying, with or without friction, on a elastic half-space or on a rigid base.

The possibility of analytical methods' application to the solution of these problems for any smooth laws of variation of non-homogeneity in depth is based on the numerical construction of transforms of the integral equations kernels referring to corresponding contact problems [1] with the subsequent analytical approximation of dual integral equations kernels, which correspond to the interaction of plate with the basis, expressions of a special type. For the construction of the approximate analytical solution of the corresponding integral equations the bilateral asymptotic method is used [2, 3], which is found to be effective here as for rigid, as well for flexible plates, in contrast to the method of orthogonal polynomials, collocation method, method of small and large parameter.

Representation of plates' flexures in rows by forms of their own fluctuations [4] is used under corresponding boundary conditions, and it makes possible to reduce the specified problems to the solution of the systems of linear algebraic equations.

The analysis of influence of various laws of non-homogeneity on depth variation, on the allocation of contact pressures under the circular plate; on its flexures; on a

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surface settlement of the functionally-graded layer outside of plate; on the meaning of the radial and tangential moments in the thin circular plate is carried out.

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Axisymmetric Bending Analysis of two Directional Functionally Graded Circular

Reza Akbari Alashti and H. Rahbari

Abstract Bending analysis of a functionally graded circular plate with clamped boundary conditions is carried out. Material properties of the plate are assumed to be functionally graded in two directions, namely in the radial direction and through the thickness of the plate obeying exponential distribution laws. Poisson's ratio is assumed to be constant throughout the plate.

Governing equations of a circular plate with material properties varying only through the thickness of the plate based on the third order shear deformation theory are employed. In order to carry out the bending analysis of the two directional functionally graded circular plate, the plate is divided in number of annular plates with material properties graded through their thickness only. Material properties of each annular plate are defined by the corresponding exponential grading rule and the radial position of its center, hence these are assumed to be constant for each plate but different from its adjacent plate. Imposing the overall boundary conditions and compatibility conditions of adjacent annular plates, the governing equations of all annular plates are obtained and solved simultaneously. The analytical results are compared with the results obtained by the finite element method and are found to be in very good agreement.

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On the Formulation Strategies of Theories for Two-dimensional Structures

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Abstract Various approaches exist to formulate general shell or plate theories. The approaches can be classified, for example, by the starting point of the derivation. This can be the well-known three-dimensional continuum mechanics equations. At present these equations are preferred by the engineers. In contrast, one can introduce à priori a two-dimensional deformable surface which is the basis for a more natural formulation of the two-dimensional governing equations. Follow [1], here we discuss the theories based on the Cosserat approach. The main attention is paid to rational theory of shells in the context of micropolar mechanics, i.e. to the micropolar or 6-parametric theory of shells [1, 2, 4, 5].

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Asymptotic Theory of Orthotropic Micropolar Elastic Bars and Plates

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Abstract In recent years, as applied to the problems of micro- and nanomechanics, theory of micropolar elastic thin shells, plates and bars, particularly anisotropic, layered and composite was developed.

In this paper in a thin rectangular area, as well as in the fine spatial domain, respectively, a boundary value problem of a plane or spatial linear theory of micropolar elasticity for anisotropic (orthotropic) bodies is considered. Supposing that the overall stress-strain state can be divided into internal and boundary layer, models of orthotropic micropolar elastic bars and plates with free rotation, with constrained rotation, with small shift rigidity are constructed by the asymptotic method [1]. The theory of micropolar orthotropic boundary layer, as well as the problem of interaction of asymptotic approximations of the internal and boundary layer problems is studied.

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Mechanics of Biological Membranes from Lattice Homogenization

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Abstract The asymptotic homogenization technique is involved to derive the effective elastic behavior of biological membranes viewed as quasi periodic beam networks at the nanoscale. Thereby, a systematic methodology is established, allowing the prediction of the overall mechanical properties of biological membranes for a given network topology, expressed as closed form expressions of the geometrical (persistence length) and mechanical nano-parameters (traction and flexion moduli of the beams). The peptidoglycan and the erythrocyte have been analyzed using this methodology, and their effective moduli are calculated and recorded versus the geometrical lattice parameters. A classification of lattices with respect to the choice of the equivalent continuum model is proposed: the Cauchy continuum and a micropolar continuum are adopted as two possible effective medium, for a given beam model (Bernoulli or Timoshenko). The relative ratio of the characteristic length of the equivalent micropolar continuum to the unit cell size determines the relevant choice of the equivalent medium. The Cauchy continuum is sufficient to model membranes in most of their configurations. The peptidoglycan network may exhibit a re-entrant hexagonal lattice, for which micropolar effects become important. The homogenized moduli give accurate results, as revealed by FE-simulations performed over the whole network. The nonlinear mechanical behavior of the hexagonal and triangular lattices is lastly calculated using a perturbation method. The homogenized constitutive law can further be used in multiscale finite element simulations of whole cells submitted to control loadings.

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Evaluation of the Mechanical Parameters of Nanotubes by Means of Nonclassical Theories of Shells

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Abstract In [1] the stiffness of bridges and cantilevers made of natural chrysotile asbestos nanotubes has been studied by means of scanning probe microscopy. The stiffness is defined as a ratio the value of local load (applied to the tube) to the value of the displacement. The nanotubes with different materials for fillings are analyzed. The experiments show that the stiffness of the tube depends on the materials for filling. The tubes with water are softer than the tubes without filling materials and the tubes filled with mercury are more rigid than tubes without filling materials. It was shown [1] that the classical theory of beam bending could not explain the experimental results, but the experimental results well agree with the Timoshenko-Reissner theory (at least qualitatively), when interlaminar shear modulus of elasticity changes for different filling materials. When additional factors such as lamination of structure and cylindrical anisotropy are taken into account the theory of Rodinova-Titaev-Chernyh (RTC) permits to obtain much more reliable results. In this work the authors also applied one more nonclassical shell theory, namely the shell theory of Paliy-Spiro (PS) developed for shells with moderate thickness. The comparison of nonclassical shell theories (RTC and PS) with experimental data and FEM calculations are presented in the report.

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Closed-form Approximate Solution for the Postbuckling Behavior of Orthotropic Shallow Shells Under Axial Compression

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Abstract The current paper deals with a closed-form approximate solution for the postbuckling behavior of an unstiffened, single-curved, orthotropic shell. As it has been shown lately for example by Diaconu and Weaver [1] who examined the case of an compressively loaded infinitely long plate as well as Mittelstedt and Schröder [2] who dealt with a plate of finite length, closed-form-type solutions offer the merit of a very high computational efficiency while delivering quite satisfying results if the structure is not loaded far beyond the linear buckling load.

In the present work the shell is assumed to be thin and of small curvature, so that the non-linear shell theory according to Donnell [3] can be applied. As loading condition the case of uniform axial compression is treated (see Fig. 1). Concerning the boundary conditions all edges are supposed to be simply supported. Additionally, geometrical imperfections in form of an initial deflection of the shell can be accounted for. Firstly, the partial differential equations which describe the compati-

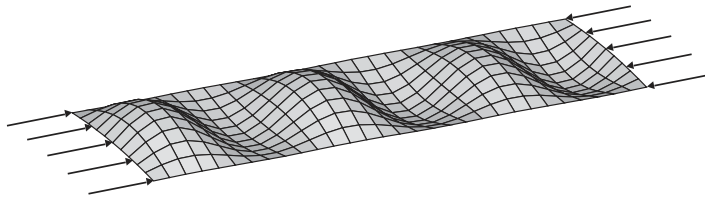


Fig. 1 Typical deformation of a compressively loaded shallow shell in the initial postbuckling state

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bility requirement concerning the in-plane strains and the equilibrium condition for a deformed shell are presented. Choosing a rather simple function of known shape but unknown amplitude for the deflection and a function of the same shape and known amplitude for the initial deflection, the compatibility requirement yields a closed-form expression for the Airy stress function. As the equilibrium condition cannot be satisfied exactly the solution procedures of Galerkin as well as Ritz are employed to obtain an approximate solution. The resulting expressions from these procedures only demand the calculation of the roots of a polynomial of third order to determine the unknown amplitude in the case of a force controlled solution whereas the force can be directly obtained in case of an amplitude controlled solution. After the force and the amplitude are known all other state variables such as stresses and displacements can be evaluated in a closed-form manner.

Due to the rather simple formulation of the deflection shape the algorithm is limited to cases where the qualitative shape of the deflection does not change significantly. On the other hand the very high computational efficiency of the described solution procedure makes it ideally suited for use in the field of optimization and preliminary design, if the applied load does not exceed the linear buckling load too much.

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Analysis of the Deformation of Multi-layered Orthotropic Cylindrical Elastic Shells Using the Direct Approach

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Abstract The paper is concerned with the deformation of thin elastic shells modeled by the direct approach. We employ the theory of *simple shells*, which was introduced by Zhilin [1, 2]. A mathematical study of the theory of simple shells is presented in [3]. In this approach, the thin shell-like bodies are modeled as deformable surfaces with a triad of rigidly rotating directors attached to each point. The model has been successfully applied in the case of plates to describe the mechanical behavior of functionally graded, viscoelastic or porous plates in [4–6].

We investigate the deformation of cylindrical shells with arbitrary shape of cross-section (not necessarily circular), and made of orthotropic materials. We consider a quite general set of constitutive equations which can describe various types of composite shells, such as multi-layered shells or reinforced shells. Moreover, our results apply also to non-homogeneous shells, since the constitutive coefficients are assumed to be arbitrary functions of the circumferential coordinate.

In this framework, we consider the following mechanical problem: find the equilibrium of cylindrical shells under the action of resultant forces and moments applied to the end edges of the cylindrical shell, and the action of given body loads (body forces and couples).

We present a general analytical solution procedure for this problem. Thus, the solution for the displacement and rotation fields is expressed in terms of the exact solutions to auxiliary boundary-value problems for ordinary differential equations. The idea of this method is taken from the three-dimensional linear elasticity: in this context, the deformation of solid cylinders made from Cosserat materials has been investigated in details, see e.g. the book [7].

Adapting the method of Ieşan [8], we investigate the deformation of cylindrical sur-

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faces with directors. We derive first the equilibrium solution of our problem for the case when the body loads are independent of the axial coordinate, but they depend only on the circumferential coordinate of the surface. Then, we consider the more general case when the body loads are polynomials in the axial coordinate (of degree m) and we obtain a solution of our equilibrium problem by the mathematical induction procedure with respect to the degree m .

We show that our solutions are in agreement with the classical results concerning the deformation of cylindrical elastic shells. Finally, we apply this general method to the case of a three-layered circular cylindrical shell, and determine the closed-form solution for the displacement and rotation fields.

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Stability Analysis of Functionally Graded Plates Subject to Thermal Loads

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Abstract Stability analysis of functionally graded ceramic-metal plates under thermal loads is presented using the first order shear deformation theory. The effective material properties of the functionally graded plates are assumed to vary through their thickness direction according to the power-law distribution of the volume fractions of the constituents. The thermal loads are assumed to be uniform, linear and non-linear distribution through-the-thickness. The derived equilibrium and buckling equations are then solved analytically for a plate with simply supported boundary conditions. Numerical examples cover the effects of the gradient index, plate aspect ratio, side-to-thickness ratio and loading type on the critical buckling for FGM plates.

Key words: Buckling; functionally graded plate; first order shear deformation theory; Thermal loading.

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Higher-order Computational Models for Analysis of Shell-like Structures

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Abstract

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Dynamic Analysis of Debonded Sandwich Plates with Flexible Core - Numerical Aspects and Simulation

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Abstract Sandwich materials are preferable for using in aircraft, airspace, naval and automotive applications due to their weight-to-strength ratio and energy absorption abilities. Most of modern sandwich panels consist of rigid face sheets and a very flexible and lightweight core commonly made of polymeric foams and non-metallic honeycomb structures. Although this type of materials is very useful, it is always accompanied with interfacial damage, so-called debonding, caused by out-of-plane loading or manufacturing imperfections. Therefore, sandwich structures must be designed so that to sustain in-service static and dynamic loads with the existing damage. In this paper the dynamic problems of sandwich plates with the local debonding effects are being discussed. The flexible type of cores under flexural motion may change the thickness as a consequence upper and lower face sheets exhibit different deformation pattern. Moreover, the severe discontinuity of mechanical properties between constituent layers yields high stresses at the core-to-face sheet interface that may initiate debonding propagations. Thereby, the classical sandwich theories based on the assumptions of anti-plane incompressible core and perfect bonding interface are not valid longer and new approaches are required. Basing on the high-order sandwich theory that assumes the classical plate-like theories for face sheets and the elasticity theory of a three dimensional medium with shear and transverse resistance for the core, the core compressibility during oscillations of perfect bonding sandwich plates can be taken into account. The complete set of governing equations for studying dynamic behavior of debonded sandwich plates must involve additional relationships obeying contact-impact conditions at a local plate zone. A model of a vibrating sandwich plate with a soft core containing partial debonding at the core-to-face sheet interface is developed based on the

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finite element method in the presented paper. The computational model is created with ABAQUS code and consists of continuum shell elements for each of the face sheets and 3-D brick elements for the core. The debonding is modeled by introducing a real gap between nodes with coinciding in-plane coordinates. The intermittent contact-impact conditions during oscillations are implemented by using interface contact properties for the detached fragments in the vibrating sandwich plate. The *hard contact* model along with the penalty algorithm for resolving the contact problem is accepted. The energy dissipation phenomenon in contactable parts between the face sheet and the core is modeled by introducing the core material model with appropriate hardening law. The explicit integration rule within ABAQUS/Explicit is used to solve the problem stated and to study the dynamic transient response of the debonded sandwich plate. The dynamic effect of the core and various parameters of the debonding zone are discussed in details.

A Best Theory Diagram for Metallic and Laminated Shells

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Abstract Among the various problems related to design and analysis of metallic and layered shells this paper focuses on the development of appropriate shell theories for a given geometry, loadings, boundary conditions as well as metallic and layered wall. It is known that classical plate theories, such as classical Theory based on Kirchhoff-Love postulates are not capable to trace accurately local stress/strain response of metallic thick structures and anisotropic shells. Laminated structures, in fact, due their intrinsic through-the-thickness anisotropy, show the so called zig-zag ZZ form of displacement field as well as interlaminar continuous IC transverse shear and normal stress fields at layers interfaces. These peculiarities of multilayered construction where summarized as C_z^0 -requirements in [1] and cannot be considered by classical plate theories. Many studies have been proposed to overcome limitation of classical theories. Higher order theories with unknown variables whose numbers is independent by the number of the constitutive layers N_l have been denoted as Equivalent Single layer models ESLM; others which kinematic is independent in each layer have been denoted as Layer-Wise models LWM. Zig-zag effect is naturally accounted for by LWM while possibilities to include ZZ effect and IC have been extensively discussed in the review papers by Carrera [1, 2]. Most of the practical problems related to composite structures demand the use of computational models, that is 3D solution as well as closed form analysis are restricted to very few cases.

By using refined computational models for layered structures the following question arise to the users:

Is there any possibility to establish the most accurate and less expensive computational model/theory for a given shell problem?

If we restrict the attention to shell case we can look at the problem by considering the diagram in Fig. 1. The x-axis shows accuracy (measured for a given

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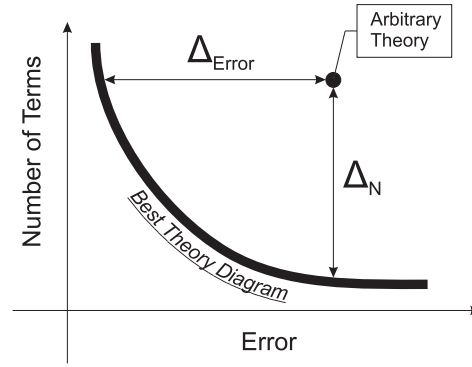


Fig. 1 Best Theory Diagram: costs vs accuracy.

stress/displacement/strain/failure variable) and the y-axis computational costs (ndof) of a given plate theory. For a given problems, geometries, lamination lay-out, loadings and boundary conditions, the theory under consideration can be represented by a point in that diagram. The following two equations can be made:

1. Is it possible to reduce the error with the same computational costs?
2. is it possible to reduce the computational costs with the same error?

To provide answer to the above questions appears extremely relevant to shell-theory users. It appears clear by Fig. 1 that to provide answer to questions 1 and 2 means to find the two corresponding points at constant error and constant computational costs, respectively. By doing that for any possible plate-model it appear clear that a Best Theory Diagram can be build which gives for each accuracy the minimum dof required or viceversa. It is concluded that BTD is a tool that provide answer to questions 1 and 2. On the basis of Carrera Unified Formulation [1, 2] and recent works [3, 4] BTD has been introduced in [5] for the case of theory for isotropic one-layer plates. This work extend the construction of BTD to shell geometry.

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On Modeling and Non-Linear Elasto-plastic Analysis of Thin Shell Structures with Deformable Junctions

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Abstract Thin-walled shell structures often consist of several regular shell elements interconnected along junctions. Pressure vessels, silos, liquid and gas storage tanks, offshore platforms, tubular towers, branching and intersecting pipelines, rockets, multi-segment underwater pressure hulls are just few examples of such irregular shell structures. In most of such multi-shells the adjacent elements are usually connected rigidly to each other. But in some cases the shell junctions may have their own mechanical properties allowing the adjacent shell elements to deform (usually rotate) one with respect to the other. Such junctions are called deformable here.

In this paper we first recall the theoretical formulation of [1] and extend it slightly at the junction by explicit account of the junction deformability in the principle of virtual work. Then in the incremental-iterative solution procedure of the non-linear boundary value problem for the whole irregular shell structure with deformable junctions is analyzed. In order to evaluate the effectiveness of such modeling of elasto-plastic shell behavior with deformable junctions in a simple one-dimensional case, an appropriate algorithm based on the computer code MINIMOD described in [2] is developed. The algorithm uses the C^1 axisymmetric displacemental finite elements applied within the elasto-plastic shell behavior [3] which is extended here to account for the deformable junctions.

Numerical tests are performed for an axisymmetric non-linear behavior of the casing of measuring device subjected to external pressure. The casing consists of the cylindrical part joined with two toroidal caps connected at boundaries with other rigid parts through welding modeled here as elastically deformable junctions. The influence of the junction stiffness on the stress distribution in the casing and on the buckling pressure of the whole composed shell structure is discussed.

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On Consistently Refined Constitutive Equations in the Geometrically Non-Linear Theory of Elastic Shells

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Abstract The general non-linear theory of shells proposed by Reissner (1974) was developed in a number of papers and partly summarized in the books by Libai and Simmonds (1998), Chróćielewski et al. (2004), and Eremeyev and Zubov (2008). This dynamically exact and kinematically unique, two-dimensional (2D), six-field shell model naturally includes the so-called drilling rotation and two transverse shear stress couples with corresponding work-conjugate transverse shear bending. These fields become of primary importance in analysis of irregular shells with kinks, branchings and intersections, when connecting shell elements with beams, columns and stiffeners, as well as in 2D formulation of singular phenomena in shells such as phase transition, crack propagation, dislocations and disclinations, etc.

In the general six-field shell model discussed here the 2D stress resultants and couples are defined by through-the-thickness integration of corresponding 3D stress distributions, while the work-conjugate strain measures are defined only on the 2D level.

As a result, the idea of consistent second approximation of the 2D elastic strain energy density proposed by Pietraszkiewicz (1976) cannot be applied. Hence, here we integrate through the shell thickness the 3D complementary energy density of the geometrically non-linear elasticity. Then we use concrete quantitative error estimates for stresses and their derivatives obtained by John (1965), which allows one to consistently refine the first approximation theory taking into account all the second-order terms. As a result of some estimation procedure, the consistent second approximation to the complementary energy density of the geometrically non-linear theory of elastic shells is constructed. Two main terms of this 2D formula constitute the consistent first approximation to the complementary energy of the elastic shell, which is energetically equivalent to the consistent first approximation to the elastic

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strain energy density of Koiter (1960). The remaining five second-order terms provide the consistent refinement of the 2D complementary energy density of the shell. Then the consistently refined constitutive equations for 2D strain measures follow from differentiation of the 2D complementary energy with regard to the appropriate work-conjugate 2D stress measures. Since these equations become linear, one can invert them for the 2D stress measures as well.

Numerical simulations of behavior of several complex shell structures, with influence of the secondary terms in the 2D complementary energy on the numerical results taken into account, will be discussed and presented during the EUROMECH Colloquium 527.

The Influence of Transverse Shear on Edge Buckling

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Abstract In a number of recent works the author and his associates have proposed a boundary-layer strategy for dealing with edge buckling of radially stretched annular thin films; certain rectangular configurations were also considered in detail. The characteristic features of the instability phenomena investigated were a short-wavelength deformation pattern concentrated near the edge of the two-dimensional domains occupied by the films. Since the analysis was carried out with the help of the von Kármán-St. Venant classical equations for thin plates with initial stresses, several subtle issues remained beyond the scope of the mathematical model adopted. It is the purpose of this contribution to return to those studies and provide a rational assessment of a number of new factors, such as the influence of transverse shear on the boundary-layer behavior reported earlier.

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Gauss-Codazzi Equations for Thin Films and Nanotubes Containing Defects

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Abstract In [1, 2] it was proposed that an appropriate shell theory is needed for the rigorous continuum modeling of carbon nanotubes containing defective structures. Understanding the behavior of defects in various slender bodies like thin films is also critical for the improvement of the reliability of many micro-electro-mechanical systems [5].

In this work we concentrate ourselves mainly on the geometrical side of such theory and treat simultaneously all the objects at hand as two-dimensional surfaces owing very nontrivial geometry in the sense of Riemann and Cartan. Using the methods described earlier in [3, 4] we show that the presence of edge dislocations leads to the existence of the torsion vector on the surface and the presence of screw dislocations leads to a nonsymmetrical second fundamental form with an additional degree of freedom being interpreted as the density of screw dislocations. For that surface the full nonlinear system of the Gauss-Codazzi equations is deduced describing how internal defective geometry of the surface fits well within the external defect-free geometry.

In the case of inextensional deformations (bending problem, isometrical transformations), i.e. when the metrics on the surface is Euclidean and there are no edge dislocations, the Gauss-Codazzi equations have very remarkable structure

$$\begin{cases} K_{11}K_{22} - K_{12}^2 = -a^2, \\ \frac{\partial K_{11}}{\partial x_2} - \frac{\partial K_{12}}{\partial x_1} = \frac{\partial a}{\partial x_1}, \\ \frac{\partial K_{22}}{\partial x_1} - \frac{\partial K_{12}}{\partial x_2} = \frac{\partial a}{\partial x_2} \end{cases}$$

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Here $K_{\alpha\beta}(x_1, x_2)$ are the components of the usual (symmetric) second fundamental form and $a(x_1, x_2)$ is the density of screw dislocations.

We hope that such a system will be of some interest from the point of view of classical differential geometry on surfaces and possibly give rise to some purely mathematical analysis of the problem.

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Model Reduction for the Numerical Simulation of Shell Dynamics

Dieter Dinkler and Jens Pontow

Abstract The numerical simulation of complex dynamic systems may be performed efficiently by applying model reduction methods [1, 2]. In this paper, the applicability of a projection-based reduction method for the evaluation of the stability and the post-buckling behaviour of imperfection-sensitive shells is investigated. Particular attention is laid on an appropriate selection of the basis vectors and the identification of critical states which are relevant for the stability of shells [3, 4].

The numerical model for the non-reduced system is based on Naghdi's shell theory [5] and a mixed formulation of the governing equations. Geometrical nonlinearities are considered as 'moderate rotations', an approximation adequate for analysing shell buckling phenomena. For the model reduction, different sets of basis vectors comprising amongst others some natural vibration modes and a Krylov sequence of modes are used. Moreover, the mode representing the difference in the state variables between a fundamental state and the corresponding critical state defined within the framework of the perturbation energy concept, cp. [3], is taken into account. The applicability of the proposed reduction method is discussed by means of a cylindrical shell under axial pressure and a spherical shell under radial pressure.

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On the Tolerance Modelling of Geometrically Nonlinear thin Periodic Plates

Łukasz Domagalski & Jarosław Jędrysiak

Thermal Stress and Strain of Solar Cells in Photovoltaic Laminates

Ulrich Eitner, Sarah Kajari-Schröder, Marc Köntges, and Holm Altenbach

Abstract A photovoltaic(PV) module is a collection of crystalline silicon solar cells embedded in a package for environmental protection [1]. This thin layered structure consists of a glass superstrate, one layer of a transparent elastomeric sheet, the electrically interconnected solar cells, a second elastomeric sheet and a polymeric back sheet [2]. As PV modules are sold with warranties of 25 years there are high reliability requirements to withstand outdoor exposure in different climates. Among the accelerated aging tests is the thermal cycling procedure where thermal loads from day-and-night cycles and from seasonal temperature changes are imitated. Modules with the standard design are commercially deployed since the 1980s and this standard concept has by now proven that it meets the reliability demands [3]. However, as further cost reductions are necessary the use of different designs [4] and alternative materials [5,6] must be carefully investigated before being industrially implemented. In the case of thermomechanical stability it is no longer sufficient to check the electrical properties of the module after accelerated testing but the creation of stresses and strains within all material layers must be quantified and well understood.

We investigate the thermal deformation of the embedded solar cells in PV module like structures both experimentally and by the help of FEM simulations. It has been shown that a stereo camera system with digital image correlation is well suited to measure the change in the distance between two adjacent solar cells in the temperature range of -40°C to 90°C [7]. In order to set up a mechanical model of the module the mechanical properties of the different bulk materials are experimentally determined or taken from literature. Two dimensional plane stress simulations are carried

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out to reproduce the experimental data of the change in the cell distance. It is found that the complexity of the mechanical model for the elastomeric sheet is highly relevant for a good agreement with the measured data. This fact is illustrated by using linear elastic, temperature-dependent linear elastic and viscoelastic material models for the transparent encapsulation sheet. Furthermore the factor for the creation of strain in the laminate is identified to be the thermal mismatch between the glass and the solar cells rather than the mismatch to the polymeric layers. For temperatures above 0°C their stiffness is too low to take effect. For temperatures below 0°C the encapsulation material enters its glass transition region and begins to constrain the cells' deformation that results from the thermal mismatch to the glass superstrate. These results enable a precise optimization of the electrical interconnection design between two solar cells to withstand thermal cycling tests and allow the evaluation of different module designs and materials with respect to thermal loading.

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On the Nonlinear Theory of Two-phase Shells

Victor A. Eremeyev

Abstract The aim of this work is to discuss the nonlinear theory of shells made of material undergoing the phase transitions (PT). The interest to such thin-walled structures is motivated by the applications of the thin films made of martensitic materials, see [1, 2] among others, and the modeling of biological membranes. The equilibrium compatibility conditions for shells with PT are given in [4–6] within the framework of 6-parametric theory of shells. The quasistatic behavior of two-phase shells is analyzed in [6, 7]. Here we present few new solutions of the boundary-value problems of shells and plates with PT.

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Nonlinear Magnetoelastic Waves in a Plate

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Abstract Nonlinear distortions caused by the propagation of intense elastic waves in construction elements may accumulate over the course of time and under certain conditions may heighten the steepness of wave edges. This, in turn, may lead to high tension and irreversible deformations in the material, resulting in local instability. On the one hand, the effects of the formation of nonlinear waves with high tension and deformation gradients are undesirable because they may lead to the destruction or plastic flow of the material, but, on the other hand, they may be useful and find application in materials processing technologies, defectoscopy, and technical diagnostics.

Effects of magnetoelasticity appear in strong magnetic fields when the induced strains have the major influence on wave and dissipative characteristics of elastic media such as rods, planes, and membranes. Anisotropy of properties is typical for boundless magnetoelastic media in a magnetic field. In a medium with finite conductivity, the magnetic field leads to the additional dissipation mechanism. Such features of magnetoelastic systems open new possibilities for practical applications.

The work is devoted to the research of nonlinear magnetoelastic waves transmission in a plane. By introducing a small parameter into the system of equations of magnetoelasticity we reduced the system to evolutionary equation with respect to axial deformation. The obtained equation represents a combination of renowned 2-dimensional model equations of Khokhlov-Zabolotskaya-Kuznetsov and Kadomtsev-Petviashvili. We investigated localized solutions of mentioned above evolutionary equation. This is due to the localized wave energy focusing in a limited segment unlike the energy of periodic waves, and it causes heavier loads in the object that in its term may cause the destruction of the object. In each particular case dependencies of parameters of localized waves both on intensity and orientation of a

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magnetic field were analyzed. The obtained results show that it is possible to control characteristics of magnetoelastic waves with the help of a magnetic field.

Effective Mechanical Properties of Closed-cell Foams Investigated with a Microstructural Model and Numerical Homogenisation

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Abstract With their cost-effective production and advantageous properties (e.g. low density, thermal conductivity, high stiffness) foams are an attractive material for an increasing number of applications. In this work the mechanical behavior of closed-cell foams is analyzed numerically. Besides the deepening of the understanding of the mechanism of deformation and failure the identification of the components of the effective elasticity tensor is the major aim of this study. Since the mechanical properties depend on the cellular microstructure a representative volume element (RVE) of that microstructure is considered. A tetrakaidecahedral cell is implemented in a finite element routine with periodical boundary conditions and a strain-energy based homogenization concept is utilized. In this concept it is assumed that a macroscopically equivalent deformation state leads to the same strain energy in a representative volume element as in a homogenous “effective” medium with yet unknown properties. The effect of imperfections, such as curved cell walls and geometry irregularities, on the effective mechanical properties is investigated and microbuckling instabilities of the cell-walls are discussed. The results are compared with literature and experimental data.

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Models of Symmetrically Assembled Three-layer Micropolar Elastic Plates

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Abstract The model of the body with layered structure (in particular the model of bodies with coatings) based on micropolar theory of elasticity played a significant role in the mechanics of multiphase media, in tribology, as well as in nanomechanics.

In the paper on the basis of hypotheses, having the asymptotic confirmation [1], depending on the values of the physical dimensionless parameters, a general theory of symmetrically assembled micropolar elastic three-layer plates with free fields of displacements and rotations, with constrained rotation, with small shift rigidity is considered. On the basis of the constructed models of three-layer micropolar elastic plates specific problem of determining the stress-strain state of three-layer rectangular and circular micropolar plates is considered. Some specific properties of the micropolar material layer plates are shown.

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What Shell Theory fits Carbon Nanotubes?

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Abstract The mechanical behavior of carbon nanotubes has been described by *bottom-up methods*, which bridge between three different scales: the microscopic scale of *molecular mechanics*; a mesoscopic scale, at which concepts from *discrete structure mechanics* apply; and the macroscopic scale of *continuous structure mechanics* (see e.g. [1] and the literature cited therein). When such methods are used, a crucial issue consists choosing what model to pick at the macroscopic scale, whether beam-like or shell-like. Both choices have been made, but a conclusive evidence about which is the fitter is still wanted, for a number of reasons. In this paper, we discuss what linearly elastic shell model would best capture the peculiarities of the mechanical response of carbon nanotubes, be they single- or multi-wall.

In the vast majority of papers where the shell choice is made, textbook theories induced from classic three-dimensional elasticity for *isotropic* materials are used; consequently, the shell response is characterized in terms of two elastic moduli. On the basis of a careful mesoscopic modeling of armchair and zigzag carbon nanotubes, we contend that it would be more appropriate to model carbon nanotubes, at the macroscopic scale, as *orthotropic* cylindrical shells.

A shell, cylindrical or not, is orthotropic when it is modeled on a surface whose tangent plane coincides with the orthotropy plane, so that the shell geometry agrees point-wise with the geometry intrinsic to the chosen type of material response. The elastic moduli of a general orthotropic shell are nine, a number variously reduced by reducing the generality of admissible deformations. For one example, stipulation of the internal kinematical constraint typical of the Kirchhoff-Love theory of shells – an assumption compatible with orthotropy leading to a theory seemingly applicable to single-wall nanotubes [2,3] – brings that number down to four. For another example, stipulation of a relaxed Kirchhoff-Love constraint allowing for thickness distention leads to a theory with seven material moduli, a theory hopefully

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fit for multi-wall nanotubes, where the deformational consequences of the inter-wall forces of van der Waals type should not be ignored. In all cases, conspicuous analytical simplifications are achieved when attention is confined to axisymmetric equilibrium problems like axial traction and torsion, the ones for which experimental tests are especially easy to set up.

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Asymptotic Analysis of Vibration and Buckling of Joint Shells

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Abstract Free shell vibrations problems and shell buckling linear problems may be reduced to eigenvalue problems for the system of linear differential equations of the 8th order, which contains the dimensionless shell thickness as a small parameter. Therefore, these boundary value problems may be solved by asymptotic methods. In the first approximation we obtain generating system of the 4th order. If we study the vibrations or buckling of two joint shells, then we have four boundary condition at the each shell edge and eight continuity conditions at the connection line for two systems of the 8th order. To solve generating eigenvalue problem for two systems of the 4th order one must choose two main boundary conditions from four boundary conditions at the each shell edge and four main continuity conditions.

Sometimes two of four main continuity conditions contain only displacement and stress-resultants of the first shell and two other conditions contain only ones of the second shell. Then the generating eigenvalue problem for two joint shells breaks up on two independent problems, corresponding to first and second shells. In this case it is possible to obtain simple formulae for eigenvalues. This asymptotic approach has been used for analysis of low frequency vibrations and buckling under action of an external pressure of the following shell structures: conical shell joint with cylindrical shell [1], two cylindrical shells of equal radius joint at an angle [2] and cylindrical shell joint with annular plate [3]. The last problem is of great importance for the theory of ring-stiffened cylindrical shells.

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Asymptotic Integration of Narrow Plates and Shells Problems

Valentina O. Finiukova and A.M. Stolyar

Abstract Derivation of narrow plates and shells model has been carried out in the paper. It has been based on the method of asymptotic integration in connection with boundary layer method and applied to some linear and non-linear static and dynamic problems. It allows to get for solution a sequence of linear or nonlinear one-dimensional initial boundary-value problems (as a result of first iteration process) and a sequence of two-dimensional linear problems (second order process) and to get the solution of given problem with high accuracy. Small parameter here is equal to the relation of the lengths of adjacent sides of the plate. It have been obtained (while dealing with classic problems) that main members of asymptotic expansions satisfy to known equations of beams theory. Some examples have been calculated in order to find the limits of asymptotics obtained.

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In-plane Strain and Stress Fields in Theories of Shearable Laminated Plates Subject to Transverse Loads

G. Formica, M. Lembo, and Paolo Podio-Guidugli

Abstract The purpose of this work is to compare the predictions of our own theory of laminated plates with some other theories of the same type. A trait common to all theories we consider, including ours, is that a Reissner-Mindlin Ansatz for the displacement field is accepted, either layer by layer [1, 2] or for the whole plate [3], so that the resulting structure is capable of transverse shearings. How to account for shearing states accurately, a central issue in the case of laminated plates, has been the subject of many investigations. What we are after here is to determine how well the in-plane strain and stress fields are described.

We concentrate on circular plates that are axi-symmetrically loaded, whose layers are made of transversely isotropic or orthotropic materials and are symmetrically located with respect to the midplane of the plate. While our theory allows for an explicit solution of this simple problem, the governing equations of the others are solved numerically.

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From 3D Non-linear Elasticity to 1D Elastic Models for Thin Walled Beams

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Abstract Geometrically, a thin-walled beam is a slender structural element whose length is much larger than the diameter of the cross-section which, on its hand, is larger than the thickness of the thin wall. This kind of beams have been used for a long time in civil and mechanical engineering and, most of all, in flight vehicle structures because of their high ratio between maximum strength and weight. Because of their slenderness thin-walled beams are quite easy to buckle and to deform and hence, in several circumstances, their study has to be conducted by means of nonlinear theories.

In this talk, starting from three-dimensional nonlinear elasticity, we rigorously derive a hierarchy of one-dimensional models for a thin-walled beam, in the spirit of what has been done by Friesecke, James and Müller in [2] for plates. The different limit models are distinguished by the different scaling of the elastic energy, which, in turn, depends on the scaling of the applied loads. More precisely, denoting by h and δ_h the length of the sides of the cross section and by ε_h the scaling factor of the bulk elastic energy, we can identify three main regimes:

- subcritical: $\delta_h/\varepsilon_h \xrightarrow{h \rightarrow 0} 0$;
- critical: $\delta_h/\varepsilon_h \xrightarrow{h \rightarrow 0} 1$;
- supercritical: $\delta_h/\varepsilon_h \xrightarrow{h \rightarrow 0} +\infty$.

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In the subcritical regime it is easy to see that, if $\varepsilon_h = O(1)$, then the limit model coincides with the nonlinear string model deduced by Acerbi, Buttazzo and Percivale in [1] for a beam of uniformly small cross-section. If instead $\lim_{h \rightarrow 0} \varepsilon_h = 0$, we obtain the energy of an inextensible string, or more precisely, since the string can grow shorter but not longer, to use a terminology introduced by Pipkin [3], the energy of an inextensible string with slack. In the critical regime, and under the additional assumption that $\lim_{h \rightarrow 0} h^2/\delta_h = 0$, we obtain a Cosserat model for a thin-walled beam. Finally, in the supercritical regime we deduce one dimensional linear/quasi-linear models for thin-walled beams.

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Spiral Piezoelectric Shells: Modeling and Applications

Anna A. Girchenko, Holm Altenbach, and Victor A. Eremeyev

Abstract In recent years technologies that use new metamaterials with chiral symmetry have been developed, see for example [1, 2]. The principal elements of such material are spiral nanofilms. For understanding the behavior of such materials we discuss the mechanical model of multilayered piezoelectric shell.

Following [3] a modal analysis of a two-layer nanofilm having piezoelectric properties is presented. We propose two finite element models for two-layer spiral nanoshells made of piezoelectric materials. The first model describes the spiral shell alone, while the second one consist of the spiral shell and the fluid environment. Models are implemented in the finite-element package ABAQUS. We take into account the rotation of the anisotropy axes for the piezoelectric materials. The analysis is performed for different types of boundary conditions for the displacements and the electric potential.

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Numerical Analysis of Static and Dynamic Deformation of Anisotropic Elastic Shells Using Various Models

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Abstract Many elements of modern structures have the form of shells with various shape, structure, and ways of fixation and are acted upon by distributed and local loads. The wide using the shell elements is attributed to striving to satisfy the requirements which are stipulated by special conditions of operation of machines, aircrafts, different constructions, and other devices. Due to complication of structural forms and structure of shell elements, necessity arises to develop a theory and methods for solving the static and dynamic problems for shells made of anisotropic inhomogeneous materials. The typical feature of the development of the theory of plates and shells is interconnection between the process of constructing the mathematical model of the given class of problems and developing the methods of solving the problems which are described by this model. This point is supported by the Kirchhoff theory of thin plates and by the Kirchhoff-Love theory of thin shells in which beside striving to describe adequately behavior of plates and shells, the object is pursued to retain the simplicity of these models to such an extent that would secured the opportunity to solve the problems using computers of that time. As the example of such interconnection, the Mushtary-Donnell-Vlasov shell theory may be noted, where the simplifications introduced into the basic equations made it possible to construct solutions for some classes of problems in sufficiently wide range what is supported by the following names of this theory: the engineering theory of shells, theory of shallow shells, theory of shells with the high changeability factor of stress state mode. This interconnection manifests itself largely at the present time when for solving the problems of shell theory of wide using are computers. In constructing the mathematical models as applied to concrete classes of shells, it is necessary to provide for all features and possibilities associated with solving a problem.

Besides the universal approaches to the solution of the problems of mechanics and mathematical physics based on the using finite difference, finite element and other discrete methods the approaches are widely used, which allow us to reduce

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the problem to ordinary differential equations basing on the approximation of the solution in other variables by analytical means.

For this reason in the present report we propose some approaches to numerical and analytical solving the problems of mechanics of shells with various structure and shape based on the classic, refined, and spatial models.

The approaches developed are used to analyze the mechanical behavior of rectangular anisotropic plates and rectangular in plan shallow variable-thickness shells under complicated boundary conditions. The calculations are performed within the framework of the classic and refined theories of shells. To study the stress-strain state and dynamic characteristics of anisotropic thick-walled cylinders with circular and noncircular cross-section, the spatial theory is applied. The solutions of noted classes of problems are of high accuracy and have the wide range of change in geometrical and mechanical parameters of the elastic bodies being considered.

Basic Concepts in the Stability Theory of Thin-walled Structures

Ardeshir Guran and Leonid P. Lebedev

Abstract Numerous specialized books and papers have been written about the subject of stability in mechanics. Most of these concentrate on methods for obtaining critical values of certain parameters and typically contain algorithms and graphs generated for describing important but very specific problems. In the present paper we take a step back and discuss the truly central notions regarding mechanical stability. Our intention is to treat the required concepts on a fairly elementary level, while simultaneously offering a bit of useful historical perspective. We also attempt to explain some common discrepancies between theoretical and practical results.

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Computational Models of Laminated Glass Plate under Transverse Static Loading

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Abstract Laminated glass with Polyvinyl Butyral (PVB) interlayer became a popular safety glass for architectural and automotive glazing applications. The very soft interlayer bonding the glass plates, however, has negligible normal stress in transverse loading and it resists mainly by shear stress. The classical laminate theory obeying the principle of the straight normals remaining straight is not valid for laminated glass. The lack of computational models frustrates the design of complex in configuration laminated glass walls for architectural glazing.

Based on the assumption that the glass layers of a laminated glass plate obey Kirchoffs classical plate theory and the PVB-interlayer transfer load by shear stress only coupling the bending of the glass layers by membrane forces, the differential equations of a triplex glass plate are derived. Using those equations and the potential strain energy, a special shell Finite Element (FE) for laminated glass is elaborated. For each of its nodes, the element has three translational, two rotational, and two additional degrees of freedom representing the shear of the interlayer between the surfaces of the glass layers.

A rectangular simply supported plate of laminated glass has been loaded by transverse load and the strain field on both free surfaces of the plate has been estimated by a digital image correlation system ARAMIS. The results are compared with the results of calculations by three computational models of the FE method. The first model has conventional 3-D brick elements with four plies of elements for each glass layer and one ply for the interlayer. The second computational model consists of continuum shell elements for each of the glass layers and cohesive elements for the interlayer corresponding to the assumptions for the special shell element deriva-

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tion. The third computational model is using the special shell elements developed here. The models have very different computational efficiency. The comparison of the models to the experimental results allows us to estimate the usefulness of the assumption made, the importance of the geometrical nonlinearity in bending, and the material nonlinearity of the hyper-elastic PVB-interlayer for laminated glass structures.

High-Frequency Free Vibrations of Plates in the Reissner's Type Theory

Elena A. Ivanova

Abstract The classic plate theory by Kirhgof allows to accurately describe the processes slowly varying by time. To solve the problems of the plate vibrations in the case of the external loads quickly varying by time the Reissner's type plate theory should be used. The Reissner's type plate theory includes three eigenfrequency spectra: one low-frequency spectrum, asymptotic order of which is $O(h)$, and two high-frequency spectra, asymptotic order of which is $O(h^{-1})$. Solving the problems of plate vibrations under the action of the slowly varying by time loads it is necessary to take into account vibrations with eigenfrequencies from high-frequency spectra. That is why the problem of plate free vibrations with eigenfrequencies, asymptotic order of which is $O(h^{-1})$, is interesting and practically important. In this paper asymptotic analysis of the equations of the Reissner's type plate theory at high-frequency free vibrations is carried out and the approximate equations of plate vibrations with eigenfrequencies of the asymptotic order $O(h^{-1})$ are proposed. Asymptotic analysis of the equations of plate free vibrations shows that behavior of the functions defining the stress-strain state of plate at high-frequency free vibrations differs from it at low-frequency free vibrations. At high-frequency free vibrations solution includes the functions which quickly vary along the space coordinates but which are not the boundary layer type functions. Because of that using of the exact equations of the Reissner's type theory in numerical procedures is difficult. Approximate equations of high-frequency free vibrations of plates, independent of quickly varying along the space coordinates functions are formulated in this paper. These equations describe vibrations with eigenfrequencies from the high-frequency spectra only, like the classic plate theory describe vibrations with eigenfrequencies from the low-frequency spectrum only.

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Cusped Shell-like Structures

George Jaiani

Abstract The present lecture is devoted to the up-dated exploratory survey of investigations concerning elastic cusped (cuspidate) shell-like structures, namely, cusped (standard and prismatic) shells, plates, and beams, cusped shell-like elastic body fluid interaction problems. It contains some non-published results as well. Under cusped shells (see, e.g., [1-3]) we understand shells whose thickness vanishes either on a part or on the whole boundary of the standard shell middle surface and prismatic shell projection. Beams are called cusped ones (see, e.g., [4,5]) if at least at one end of the beams the area of its cross-section vanishes.

Mathematically the corresponding problems lead to non-classical, in general, boundary value and initial-boundary value problems for governing degenerate elliptic and hyperbolic systems in static and dynamical cases, respectively. Study of cusped shell-like bodies is characterized by arising concentrated along cusped edges and points forces and by dependence of well-posedness of boundary conditions in displacements along cusped edges on geometry of sharpness of edges. At present we have sufficiently complete theory of elastic cusped prismatic shells and beams but study of cusped standard shells of general form remains topical.

In Section I geometry of structures under consideration is analyzed.

Section 2 deals with the construction of hierarchical models of cusped elastic shells, mainly, prismatic ones, cusped elastic beams, and shallow fluids.

Section 3 deals with the investigation of hierarchical models of cusped elastic prismatic shells, namely, with explicit solutions for cusped elastic prismatic shell-like bodies with projections of special forms (half-plane, half-strip, angle, etc.); variational formulation of the basic 3D problems for elastic prismatic shell-like bodies; approximating function spaces; variational formulation in particular spaces; existence and uniqueness theorems; convergence results; derivation of the basic system of two-dimensional models by means of general systems and the Legendre polyno-

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mials (Vekua's system); existence and uniqueness theorems for the governing system of degenerate equations of cusped elastic prismatic shells in the N^{th} hierarchical model). Cusped Kirchhoff-Love plates are discussed as well.

Section 4 deals with the investigation of hierarchical models of cusped beams, namely with explicit solutions of problems for cusped beams in particular cases; variational formulation of the basic three-dimensional problems for beam-like elastic bodies; approximating function spaces; existence results; convergence results. Moreover, cusped Euler-Bernoulli beams (properties of the general solution of the degenerate Euler-Bernoulli equation; solution of boundary value problems, vibration and dynamical problems) are discussed as well.

Section 5 deals with relations of hierarchical models of cusped elastic shells and beams to three-dimensional models [6].

In Section 6 cusped elastic prismatic shell 3D and 2D fluid structure interaction problems are discussed.

The lecture is endowed with a wide bibliography connected with topics under consideration.

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Asymptotic Analysis of Initial Value Problems for Thin Walled Structures

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Abstract

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Application of Genetic Algorithms to the Identification of Parameters and Shape Optimization of the Nonlinearly Elastic Corrugated Membranes

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Abstract Corrugated membranes are extremely important structural parts of a great number of devices, highly sensitive (high-precision) pressure sensors in particular. In engineering of corrugated shapes different factors could be taken into account as crucial. Among them - membrane's work without buckling, buckling for the prescribed load, flatness of the membrane characteristic - dependence of the applied pressure on the liquid or gas volume, a lack of the plastic deformation and so on.

In this paper an approach to different problems of optimization and design using genetic algorithm is proposed. To model elastic behavior of the membrane the non-linear equations based on the Kirchhoff-Love hypothesis and not imposed constraints on the shallowness of the membrane's shape are used. Several multi-parameter families of the axially symmetric "design", i.e. functions of the shape that describe the dependence of the middle surface prominence on the membrane radius, are considered. These designs were obtained due to special modifications of spherical dome, conical membrane and their integration. The value of the linear section length of the membrane characteristic served as an optimality criterion.

The paper presents modified genetic algorithm, describes refinement mechanism of its settings and observes examples of the designs, which are optimal in the specified shape classes.

The construction of the mathematical models of thin-walled structures is frequently based on the material parameters (e.g. Young's modulus and Poisson's ratio) of three-dimensional materials. However, the production process of the materials for corrugated membranes as well as the membranes themselves can modify these parameters. Therefore the problem of the material parameters identification for thin-walled structures is still vital. Some approaches to its statement and solution by means of developed algorithm on the basis of the practicable experimental data on a membrane functioning under natural operation conditions was demonstrated for different (both simple and generalized) models of material behavior.

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Biological and Synthetic Membranes: Modeling and Experimental Methodologies

Rasa Kazakevičiūtė-Makovska and Holger Steeb

Abstract Theories of two-dimensional continua have long been used to model diverse phenomena ranging from bioelasticity and fluid capillarity to elasticity of biological and synthetic membranes and mechanics of certain structural networks. Although general theories for such continua have long been available (see the recent review paper [1]), specific experimentally-determined constitutive relations for many membranes are not available or not sufficiently tested. Also, the performance of many biological membranes under specific loading conditions is not well-understood.

There are different methodologies to determine constitutive equations in continuum mechanics of three-dimensional bodies and specific forms of the relevant response functions can be determined directly from selected well-designed (homogeneous) tests. As a rule, these methods are based on specific experimental programs and certain geometries of the specimens [2]. In the case of synthetic materials, the samples may be fabricated in any required geometry. In contrast, the size and shape of biological and even certain synthetic membranes is dictated by nature or their intended use [3, 4]. In effect, the choice of specimen geometries and possible testing methods are very limited. Therefore, there is a need to identify classes of experiments that allow a direct determination of constitutive relations while preserving the natural geometry of such membranes.

In this work, we shall discuss how specific forms of response functions for biological and synthetic membranes can be identified by direct and indirect determinations of the distortion and area dilatation of the membrane surface together with its curvature geometry. The considered methodologies form the basis for the development of the phenomenological constitutive models. While such models are able to fit the mechanical data, they are limited in that they cannot elucidate the underlying physical and biological mechanisms of the macroscopic behavior of the membranes. Accordingly, we shall next discuss the structural-based constitutive models

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which offer insights into the structure, function and mechanics of the considered classes of biological and synthetic membranes. To illustrate the basic concepts and main results, we shall discuss the shape-property relationships of red blood cells, the mechanisms of cell division, the shapes and growth of aneurysms, and some other problems from the field of mechanics of natural and synthetic membranes.

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Unbending of Curved Tube by Internal Pressure

Alexei M. Kolesnikov

Abstract A membrane that is a sector of a torus is called a curved tube. The problem of pure bending of pressurized curved tubes is considered within the framework of the nonlinear shell theory by A. Libai and J.G. Simmonds [1] and L.M. Zubov [2]. In [1, 2] the approach is proposed to solve the problem of pure bending. The approach allows us to decompose the deformation into two parts: an in-plane deformation of meridional cross section, plus a rigid rotation of each of these meridional planes about some axis by linearly varying angles. In this case the equilibrium equations are reduced to the ordinary differential equations.

This approach can be used to solve the pure bending problem of a straight tube (cylindrical membrane) [2, 3]. The change of curvature is the result of the application of the bending moments to a straight tube. The feature of curved tube is the effect of unbending under internal pressure in the absence of the bending moments. This phenomenon is investigated in the presented work.

The equilibrium equations are obtained for curved tubes of arbitrary cross section. They are solved by numeric methods.

We assume for simple analysis that the thickness is a constant, the cross section is a circle and the curved tube is made of neo-Hookean or Mooney-Rivlin material.

In Fig. 1 the dependencies between the relative curvature B^* and dimensionless pressure P^* are presented for neo-Hookean material. The numerical results show that the initial curvature has a small effect to these dependencies. The curves $B^* - p^*$ has a maximum. This can mean the instability of curved tube.

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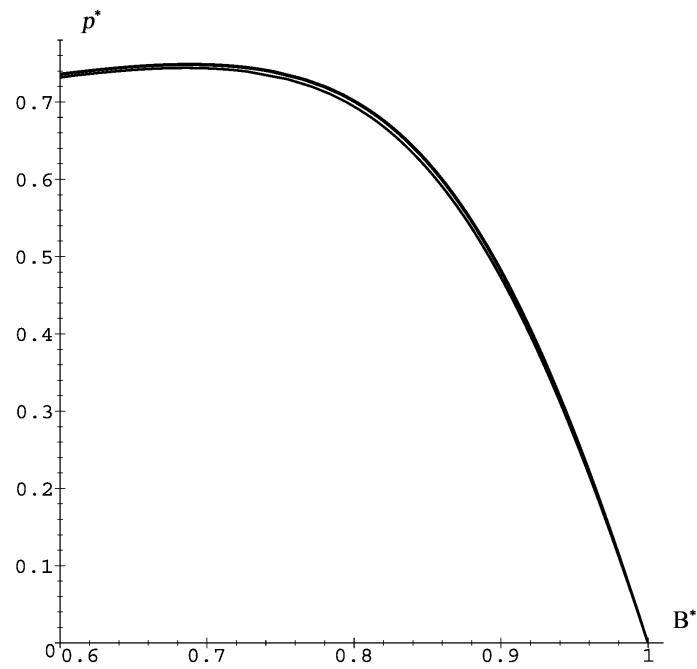


Fig. 1 Curvature-pressure.

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Multi-scale Modeling of Structured Shells Through Computational Homogenization

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Advances in Quadrilateral Shell Elements with Drilling Degrees of Freedom

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Abstract The development of analysis procedures for shell structures represents one of the most challenging tasks of finite element research. Four-node quadrilateral shell elements with one integration point in the mid-surface are known to be robust, efficient and well suited for nearly all applications in that field (e.g. [1–3]). The incorporation of the required hourglass stabilization is discussed widely in literature, and the resulting procedures frequently rely on multifield variational principles. However, in that case at least some terms require a four point integration causing computational effort. To overcome this drawback new assumed strain interpolations are introduced in our proposal in order to avoid any Gaussian quadrature at all since all parts of the entire stiffness matrix can be evaluated analytically. Further, the inclusion of in-plane rotational degrees of freedom, usually referred to as drilling degrees of freedom, pose an important issue in the derivation of general purpose shell elements. They are activated in arbitrary beam to shell connections, where classical five-parameter formulations result in singular stiffness matrices. Alternative methods to introduce drilling degrees of freedom are either based on special shape functions (e.g. Allman rotations [4, 5]) or on a modified functional initiated by Hughes and Brezzi [6] that enables the usage of low order interpolations. Our new proposal is based on a functional that avoids in contrast to [6] any mathematical regularization and is described in [7]. There, the micropolar theory of elasticity (Cosserat’s theory of elasticity) is used as starting point of the developments and the modified weak form is well suited for the derivation of membrane elements with drilling degrees of freedom. With an extended Hu-Washizu principle

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effective strain interpolations are introduced similar to methods of selective integration.

The stiffness matrix for out of plane bending is derived according to the Mindlin-Reissner theory and, therefore, is valid for thin to moderately thick shells. Here, the satisfaction of the Kirchhoff patch test is a major issue to ensure the thin plate limit, where a method similar to the well known Bathe and Dvorkin formulation [8] is used.

After all, a new and highly efficient quadrilateral Mindlin-Reissner shell element is proposed consisting of six nodal degrees of freedom. The approach is based on flat element configurations, where the membrane and the plate bending properties are decoupled. For warped element geometries a rigid body projection according to [9] is used. The entire stiffness matrix can be evaluated analytically and, thus, any Gaussian quadrature is avoided and no artificial hourglass stabilization is required. Further, no internal degrees of freedom (non-conforming modes) are necessary which would have to be condensed at element level resulting in extra computational effort. The element's performance is excellent in rectangular element configurations and the trapezoidal locking for in-plane bending type loadings is comparable to other efficient shell elements. The generalization of the linear element formulation to geometrical and physical nonlinearities is straightforward if a corotational updated Lagrangian procedure is used, and the resulting finite element is well suited for explicit dynamics where robustness and efficiency are of major concern.

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On the Use of a new Concept of Sampling Surfaces in Shell Theory

Gennady M. Kulikov and S.V. Plotnikova

It is well-known that a conventional way for developing the higher-order shell theories accounting for thickness stretching is to utilize either quadratic or cubic series expansions in the thickness coordinate and to choose as unknowns the generalized displacements of the shell midsurface. In the present work, the $3N$ -parameter shell theory is developed using a new concept of sampling surfaces (S-surfaces). We introduce N S-surfaces inside the shell body and choose $3N$ displacements with correspondence to these surfaces as fundamental shell unknowns, where $N \geq 3$. Such choice of displacements with the consequent use of Lagrange polynomials of the $(N-1)$ th order in the thickness direction permits one to represent governing equations of the shell theory in a very compact form and to derive strain-displacement relations, which exactly represent all rigid-body shell motions in convected curvilinear coordinate system. Moreover, the proposed higher-order shell theory admits the use of complete 3D constitutive equations. Note that in recent works [1,2], the 9-, 12- and 15-parameter shell theories have been developed. In these higher-order shell theories, respectively 3, 4 and 5 equally located S-surfaces are employed. Herein, a general case with N not equally located S-surfaces is studied.

Consider a thick shell whose reference surface is referred to the convected curvilinear coordinates θ^1 and θ^2 , whereas the coordinate θ^3 is oriented along the unit vector $\mathbf{a}^3 = \mathbf{e}_3$ normal to its surface. The displacement field is approximated in the thickness direction according to

$$\mathbf{u} = \sum_I L_I^{N-1} \mathbf{u}^I, \quad \mathbf{u}^I = u_i^I \mathbf{a}^i,$$

where $\mathbf{a}^i$ are the basis vectors of the reference surface; $\mathbf{u}^I(\theta^1, \theta^2)$ are the displacement vectors of S-surfaces; $L_I^{N-1}(\theta^3)$ are the Lagrange polynomials of the $(N-1)$ th order such that $L_I^{N-1}(z^J) = 1$ for $J = I$ and $L_I^{N-1}(z^J) = 0$ for $J \neq I$; z^I are the trans-

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verse coordinates of S-surfaces; indices I, J identify the belonging of any quantity to S-surfaces and take the values $1, 2, \dots, N$. Taking into account that displacement vectors of S-surfaces are resolved in the reference surface frame the proposed higher-order shell formulation is promising for developing the exact geometry (EG) solid-shell elements. The term EG reflects the fact that the parametrization of the reference surface is known and, therefore, coefficients of the first and second fundamental forms are taken exactly at element nodes.

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Three-point-bending of Plate Strips with Frictional Supports and Finite Rotations. Closed-form Solution, Finite Element Analysis and Experimental Study

Andreas Kutschke, Holm Altenbach, Konstantin Naumenko, and Walter Fischer

Abstract Large rotations three-point bending tests can be applied to examine the behavior of thin metal sheets composed by an adhesive layer. To analyze the test results comprehensible formulae for the maximum deflection and stress resultants as functions of the applied force are required.

In this study we apply the inextensible rod theory to derive governing equations of the plate strip under consideration of large cross section rotations and frictional sliding in supports. A simple semi-analytical solution for the maximum deflection as a function of the applied force is presented. The results are compared with experimental data of the deflection controlled three point bending. For short strips the solution agrees well with experimental data in the whole range of deflections from zero value to the value that corresponds to the u-formed shape of the beam. For long strips the own weight must be taken into account to reproduce the deflection accurately. To compute the deflection of long strips we perform the non-linear finite element analysis with contact friction sliding on supports.

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Nonlinear Strain Gradient Elastic Thin Shallow Shells

K.A. Lazopoulos and A.K. Lazopoulos

Abstract The governing equilibrium equations for strain gradient elastic thin shallow shells are derived, considering non-linear strains and linear constitutive strain gradient elastic relations. Adopting Kirchhoff's theory of thin shallow structures, the equilibrium equations, along with the boundary conditions, are formulated through a variational procedure. It turns out that new terms are introduced, indicating the importance of the cross-section area in bending of thin plates. Those terms are missing from the existing strain gradient shallow thin shell theories. Those terms highly increase the stiffness of the structures. When the curvature of the shallow shell becomes zero, the governing equilibrium equations for the plates are derived.

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Invariant-Based Geometrically Nonlinear Formulation of a Triangular Finite Element of Laminated Shells

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Abstract A non-standard approach is proposed to develop a simple and computationally effective triangular finite element applicable to geometrically nonlinear analysis of composite shells. The approach is based on the natural components of the stress and strain tensors and their invariants which allow one to express the strain energy of the shell in a compact form without coordinate transformations. The natural components of the tensors are referred to fibers oriented along the triangle edges. An advantage of using the natural strains is that it suffices to approximate one-dimensional functions rather than two-dimensional strain fields over the elemental area. To this end, analytical solutions of auxiliary beam bending problems are used. The validity and accuracy of the approach proposed are verified using benchmark solutions and numerical data available in the literature.

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The Effect of the Tangential Loads on the Bending of Elastic Plate

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Abstract In this work, the problem of bending of a semi-plate under the action of tangential loads is considered on the base of the classical theory, the theory of Reissner-Hencky-Mindlin by Vasilyev variant, and the theory of Ambartsumyan. The comparisons between deflections, crosscutting forces and moments by refined theory of first-order and by higher-order refined theory are then obtained.

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A Variationally Consistent Cosserat Theory

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Abstract By introducing the concept of a particle into a variational principle, it is shown how a micromorphic or micropolar Cosserat theory can be derived from the classical equations of continuum mechanics. The geometrically nonlinear strain measures, the equilibrium equations and the constitutive equations can be derived in a variationally consistent way, and it is shown that there is a close relationship between Cosserat theories and representative volume elements.

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Consistency Issues in Shell Elements for Geometrically Nonlinear Problems

Teodoro Merlini and Marco Morandini

Abstract This paper aims at highlighting and discussing some singular concepts and non-standard practices in the numerical solution of (geometrically) nonlinear shell problems. These distinctive aspects have been elaborated along with the recent development of an original formulation and numerical approximation for shell mechanics [6, 7] and are now deepened here as stand alone hints for workers in this field.

Thorough micropolar setting of the surface mechanics

It should be noted that shells - contrary to beams - are hybrid structured solids: in fact a shell, though capable of withstanding bending and torquing, is essentially a non-polar medium in its tangent plane. So a claim of withstanding drilling rotations by material constitutive means looks questionable. Rather, the drilling rotation is a redundant degree-of-freedom whose definition would entail an extra stress field.

Our approach to achieve a full micropolar setting of the 2D shell mechanics descends from the 3D solid mechanics formulated in a micropolar description context. In a displacement-based variational environment for hyperelastic nonpolar media, the internal contribution to the total virtual work is cast in the internally constrained form

$$\Pi_{\text{int}\delta} = \int_V \delta(w + \langle \hat{\mathbf{t}} \times, \boldsymbol{\Phi}^T \mathbf{F} \rangle) dV, \quad (1)$$

where the polar decomposition of the deformation gradient - that actually defines the material rotation - is either introduced as an appended constraint [8] or descends from a sound constitutive characterization [2, 5] (here $w = w(\boldsymbol{\epsilon}^S)$, $\boldsymbol{\epsilon}^S = (\boldsymbol{\Phi}^T \mathbf{F} - \mathbf{I})^S$ and $\hat{\mathbf{T}}^S = \partial w / \partial \boldsymbol{\epsilon}^S$ are the symmetric strain and stress parameters, $\hat{\mathbf{T}} = \check{\mathbf{T}}^S + \hat{\mathbf{t}} \times$ the Biot stress tensor, $\hat{\mathbf{t}}$ the workless Biot-axial vector). The primary unknown fields in Eq. (1) are displacement, rotation and Biot-axial. Integration over the thickness of

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Eq. (1) in a finite-element-like sense requires the choice of approximate fields for both the kinematic variables (displacement and rotation) and the Biot-axial stress vector. The parameters of the latter approximate field hold as primary unknown fields of the 2D shell mechanics besides the appropriate kinematical unknown fields. The shell constitutive equations relate such 2D unknown fields to the relevant work-conjugate fields in a coupled way and allow to solve nonlinear shell problems formulated in a thorough micropolar 2D description context, including any drilling rotation.

Consistent approximation of the element kinematic field

It is common practice in finite-element technology to interpolate positions in the Euclidean vector space by looking for the weighted average position from the equation

$$\sum_{J=1}^N W_J (\mathbf{x} - \mathbf{x}_J) = \mathbf{0} \quad (2)$$

Solution of Eq. (2) for normalized weights yields $\mathbf{x} = \sum W_J \mathbf{x}_J$ in closed form. This interpolation scheme allows to build the metric within the element in the reference configuration and to recover, in any current configuration, the position $\mathbf{x}'$ and the deformed metric. Thereby, in the Euclidean linear space, the unknown displacement $\mathbf{u} = \mathbf{x}' - \mathbf{x}$ can be straightforwardly interpolated among the nodal displacements as $\mathbf{u} = \sum W_J \mathbf{u}_J$, leading to the isoparametric element concept.

The same scheme could be used to interpolate orientations, which are described by the special orthogonal group $SO(3)$. The most simple and straight approach is to start from the bottom: exploit the closed form $\boldsymbol{\varphi} = \sum W_J \boldsymbol{\varphi}_J$ to interpolate the rotation vector, build a rotation tensor $\boldsymbol{\Phi} = \exp(\boldsymbol{\varphi} \times)$ and recover the current orientation $\boldsymbol{\alpha}' = \boldsymbol{\Phi} \boldsymbol{\alpha}$ from the reference orientation $\boldsymbol{\alpha}$. This approach will not yield in general a weighted average orientation in the sense of Eq. (2). Moreover, this scheme is not frame-invariant [4].

A consistent approach, instead, should start from the multiplicative analogue of Eq. (2) for the orientations [4], i.e.

$$\sum_{J=1}^N W_J \log(\boldsymbol{\alpha} \boldsymbol{\alpha}_J^T) = \mathbf{0} \quad (3)$$

The implicit nonlinear Eq. (3) does not yield in general the sought weighted average orientation $\boldsymbol{\alpha}$ in closed form and must be solved numerically [4]. Then, the angular curvatures are obtained as well, and the local rotations and angular strains are recovered by comparing the interpolated quantities in the current and in the reference configurations. This leads to the correct equivalent of an isoparametric element for rotations.

The proposed interpolation scheme is frame-invariant and path-independent at the same time - a distinctive feature for a robust finite element. Eq. (3) averages relative orientations (just as Eq. (2) averages the relative distances), and this fact ensures frame invariance. The fact of interpolating the orientations instead of rotations ensures path independence.

Variational formulation: discrete approximation and consistent linearization

The finite-element incremental solution of nonlinear boundary value problems in continuum mechanics descends from variational principles that are typically brought to discrete and linearized forms. Discrete approximation and linearization are separate processes and should yield the same result independently of the sequence of the operations. Usually, linearization is performed on the previously approximated model and might give different results w.r.t. the reverse order if some simplification is introduced when linearizing highly nonlinear interpolation schemes.

In most theories on structured solids like shells, rotations are assumed as primary unknowns. Rotations are intrinsically nonlinear functions of the relevant independent parameters and a consistent interpolation of the rotation field is highly nonlinear in the global discrete unknowns. In such circumstances, the linearization of the discrete form of a variational principle becomes a quite hard and very insidious task. We find it much more easy and convenient to first linearize independently the variational functional on one side and the interpolation algorithm on the other side, and then to apply the discrete approximation to both the functional and the incremental functional.

Once the subtle linearization process is performed once for all, the linearized variational principle is ready for any kind of finite-element approximation and can be safely used with every discrete representation one would work with - for example, a higher order shell element with mid-side nodes.

Consistent modeling of the displacement field on curved surfaces

During the evolution of a nonlinear shell analysis, the surface particles undergo *rototranslations*. By a commonly used parameterization, the rototranslation is decomposed into two separate, independent motions - displacement and rotation - and the latter is in turn parameterized by means of the eigenvector of the rotation tensor. However, the rototranslation can also be represented as an orthogonal dual tensor, namely the exponential map of a skew-symmetric dual tensor built on the helix dual vector. So, the helix is the natural eigenvector of the rototranslation. This fact suggests the helicoidal parameterization of motion as a rightful alternative parameterization, endowed with a tangent space controlled by a differential helix [1]. Thereby, displacements and rotations become coupled together into an integral kinematic quantity.

The helicoidal parameterization of the rototranslation can be applied as well to the parameterization of the geometry of a shell, both in the reference and in the deformed configurations. This leads to the so-called helicoidal modeling [3], whose tangent space is controlled by a dual curvature tensor - in contrast to the classical tangent space proper of the Euclidean modeling.

The helicoidal modeling has been resorted to for building a working shell element [6, 7] (and also in the 3D mechanics across the thickness to recover the shell constitutive equations [6]). Since rototranslations possess all the properties of the rotations, the above issues concerning interpolation, linearization and discrete approximation apply as well to the rototranslation fields. So, adopting the helicoidal modeling in shell finite elements does not introduce significant complications - rather in a sense, it simplifies things, as mere displacements are discarded in favor of one

integral rotation-type field.

The advantage of modeling helicoidally a curved surface becomes evident in the lowest-order shell elements. *Lowestorder* elements rely by nature on the tangent space of the underlying modeling: so, in a Euclidean context they are *flat* elements - e.g. 2D or 3D solid elements and membrane elements. The lowest-order helicoidal shell element, instead, is a truly curved element, featuring orientation-driven positions: i.e. the nodal orientations control both the internal rotations and the internal displacements. Numerical tests evidence that the element does not suffer from locking [7]. This welcome behavior is likely to be ascribed to the consistent multiplicative interpolation of the integral kinematic field, based on the extension of Eq. (3) to accommodate oriento-positions in a thorough helicoidal-modeling context.

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On Dynamic Modelling of thin Skeletal Annular Plates

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Theory of Thin Adaptive Laminated Shells Based on Magnetorheological Materials and its Application in Problems on Vibration Suppression

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Abstract This paper proposes the theory of thin adaptive shells based on magnetorheological (MR) materials which physical properties such as viscosity and shear modulus can vary when subjected to different magnetic fields. The laminated non-circular cylindrical shell like sandwich formed by embedding MR materials in between elastic layers is the subject of this investigation. Thickness as well as physical properties of the MR layers are assumed to be functions of the curvilinear coordinates on the middle surface of the shell.

In the capacity of the MR material the magnetorheological elastomer (MRE) consisting of magnetic particles in deformed polymer matrix was considered. As matrix for MRE a natural inorganic polymer in synthetic oil was used, as a filler - particles of carbonyl iron. Dependence of components of complex modulus of shear G' and G'' for MRE on induction of the magnetic field B was determined by means of rheometer "Physica MCR 301" by Anton Paar.

Based on both the assumptions of the generalized kinematic hypothesis of Timoshenko for whole sandwich [1] and our experimental data for MRE, we derive a system of twelve differential equations with complex variable coefficients describing motion of the shell. When vibrations occur with only minor sizes of wave length this system is reduced to the system of three equations.

As an example, free and forced vibrations of the three-layered cylinder containing interlayer MRE have been studied at fixed viscoelastic characteristics of MRE, and then, using the multiple-scale method, the reaction of the shell to the non-stationary signal of magnetic field resulting in abrupt change of MRE characteristics has been

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investigated. In addition, applying the asymptotic approaches developed in book [2], the special cases of localized vibrations and suppression of running wave packets in the shell under external non-stationary magnetic field have been analyzed.

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On the Constitutive Equations of Shells and Plates at the Nanoscale

Nikita F. Morozov, Holm Altenbach, and Victor A. Eremeyev

Abstract The aim of this work is analysis of the influence of surface effects on the mechanical characteristics of materials at the nanoscale. The surface effects play significant role for such nanomaterials as nanofilms, nanotubes, fullerenes, etc. The mechanical models of these thin-walled structures are usually based on the two-dimensional theories of plates and shells. In contrast to the mechanics of structures at the macroscale the constitutive equations for the nanosized shells have to be modified taking into account the surface phenomena.

Within the framework of 6-parametric or micropolar shell theory [1,2] we discuss the influence of surface stresses on the stiffness parameters of a shell. The considered theory of shell is free from the kinematical hypothesis or other reduction procedures used in the Kirchhoff-Love or Mindlin-Reissner theories of plates and shells. The elastic and viscoelastic micropolar shells with surface stresses are considered.

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On the Reconstruction of Inhomogeneous Initial Stresses in Plates

Rostislav Nedin and Alexander Vatulyan

Abstract The identification problem of initial self-balanced stresses in plates is considered using acoustic data. The influence of the non-homogeneous initial stresses, i.e. pre-stresses, on the frequency response function is analyzed. The weak statement of the reconstruction problem is presented. On the base of the weak statement, the iterative process is proposed. Each iteration consist of the solving of direct boundary-value problem and the integral Fredholm's equation of first kind with a continuous kernel. The integral equation allows to refine the initial stresses. The result of calculations are discussed for in-plane and out-of-plane vibrations of thin plates. The preferable loads and frequency range are given for that the reconstruction of the initial stresses is most efficient.

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Variational Formulation of Strain-Gradient Viscoplasticity Theory for Geometrically Nonlinear Shell Model

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Abstract This study derives a variational formulation for a geometrically nonlinear shell model using the theory of strain-gradient viscoplasticity proposed by Anand et al. (2008). The shell theory proposed by Simo (1989), in which the shell is regarded as an inextensible one-director Cosserat surface, is used to capture large deformation and rotation. The kinetic energy is assumed to be composed of the conventional and internal kinetic energy. The internal energy is described phenomenologically in terms of the equivalent plastic strain in order to capture the heterogeneity of plastic flow. Herein the microscopic density is assumed to be related to the macroscopic one through a microscopic-inertia parameter. The equivalent plastic strain in the cross section is assumed to be distributed linearly over shell thickness. With such assumptions, the macroscopic force and -momentum balances and microscopic force and -momentum balances including inertia effects are derived. The performance of the proposed formulation is illustrated through numerical simulations. A parameter study to find the microscopic-inertia parameter is carried out. At last, suitable microscopic boundary conditions and dynamic effects are discussed through comparison with the conventional plasticity.

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On the Convergence of the Iteration Method when Solving a Timoshenko System of Equations for a Plate

Jemal Peradze

Abstract We consider the boundary value problem

$$\begin{aligned}u'' + \frac{1}{2} (w'^2)' + p(x) &= 0, \\k_0^2 \frac{Eh}{2(1+\nu)} (w'' + \psi') + \frac{Eh}{1-\nu^2} \left[\left(u' + \frac{1}{2} w'^2 \right) w' \right]' + q(x) &= 0, \\ \frac{h^2}{6(1-\nu)} \psi'' - k_0^2 (w' + \psi) &= 0, \\ 0 < x < 1, \\ u(0) = u(1) = 0, \quad w(0) = w(1) = 0, \quad \psi'(0) = \psi'(1) &= 0,\end{aligned}$$

which describes the static behavior of a plate. The functions u and ψ are expressed through the function w for which we write an integro-differential equation with a boundary condition.

The application of the Galerkin method to the obtained problem leads to the nonlinear system of algebraic equations which is solved by iteration. The conditions of the iteration process convergence are established and the convergence rate is derived.

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On the Refined Resultant Thermomechanics of Shells with Singular Surface Curves

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Abstract The global and local resultant 2D balance laws of mass, linear and angular momenta, energy as well as entropy inequality are derived for irregular shell structures, with corresponding continuity (jump) conditions provided along stationary and moving singular surface curves. It is shown, in particular, that the resultant 2D stress power cannot be expressed exactly through the resultant 2D stress and 2D work-conjugate strain measures alone. Hence, an additional stress power called an interstitial working is added to the resultant 2D balance of energy to make it an exact implication of 3D energy balance as well. Stationary singular surface curves model geometric and material irregularities such as folds, branchings, intersections, technological junctions, abrupt changes of shell thickness or material constants, shell-to-beam connections, etc. Moving singular surface curves model such surface phenomena as wave propagation, phase boundaries, strain localization, dislocations and disclinations, cracks, etc. Two versions of the resultant 2D shell thermomechanics are discussed: with one mean surface temperature, and with an additional surface temperature deviation as independent thermal variables. The restrictions imposed by our resultant 2D entropy inequality on the forms of 2D constitutive equations of thermoelastic shells and of viscous shells with heat conduction allow as to account for some longer-range spatial interactions as well.

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Asymptotically Accurate Nonlinear Analysis of Orthotropic Hyperelastic Shells

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Abstract The work aims at dimensional reduction of orthotropic hyperelastic shells in an asymptotically accurate manner. The problem is both geometrically and materially nonlinear. The geometric nonlinearity is handled by allowing for finite displacements and rotations along generalized three-dimensional (3D) warping while the material nonlinearity is incorporated through an orthotropic hyperelastic material model. The development, based on the Variational Asymptotic Method (VAM) with moderate strains and very small thickness-to-wavelength ratio as small parameters, begins with nonlinear elasticity and mathematically splits the analysis into a one-dimensional (1D) through-the-thickness analysis and a two-dimensional (2D) shell analysis. The major contribution of this paper is the closed form nonlinear and non-classical 1D analysis which provides constitutive relation between the generalized 2D strain and stress tensors for the shell analysis and a set of recovery relations to asymptotically express the 3D displacement, strain and stress fields. Analytical expressions are derived for warping functions and stiffness coefficients. This is the first attempt to integrate an analytical work as asymptotically accurate nonlinear elastic constitutive relation for compressible orthotropic hyperelastic model with generalized 2D finite element analysis of shell mid-surfaces and to provide accurate 3D results (displacements, strains and stresses) in the most computationally efficient manner possible. A unified software package VAMNLM (Variational Asymptotic Method applied to Nonlinear Material models) is developed. Reliability and utility of the present non-classical shell theory are demonstrated through comparisons with 3D finite element based results for a few test cases.

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A new Theory of Dynamic Behavior of Spatially Curved Thin-walled Beams of Open Cross-section

Yuri A. Rossikhin and Marina V. Shitikova

Abstract The dynamic behavior of elastic spatially curved open section beams with axial precompression has been analyzed. The theory of discontinuities and the method of ray expansions, which allow one to find the desired fields behind the fronts of the transient waves in terms of discontinuities in time-derivatives of the values to be found, are used as the methods of solution for short-time dynamic processes. Using these methods, it is possible to obtain from the three-dimensional equations of the theory of elasticity the recurrent relationships for determining the discontinuities in arbitrary order time-derivatives of the desired values, which, in contrast to the Timoshenko-like theories, do not involve shear coefficients depending on geometrical parameters of thin-walled beams of open section, resulting in the fact that the velocity of the transient shear wave coincides with that of the transverse wave in a three-dimensional elastic media, in contrast to the Timoshenko-like theories, according to which several transverse waves propagate, as a rule, with the velocities dependent on the geometrical parameters of the beam. As for the longitudinal wave, then in both theories it propagates with the velocity of the longitudinal wave in a thin elastic rod.

As an example, it is considered the normal impact of a steel rod with a rounded end upon a steel arch, representing itself a channel-beam curved along an arc of the circumference. The time-dependence of the contact force is found and analyzed graphically at different levels of the axial compression force.

Keywords: Axially precompressed thin-walled beam of open section, theory of discontinuities, ray expansions, normal impact

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Mathematical Models of Micropolar Elastic Thin Shells

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Abstract One of the actual problems of micropolar (asymmetrical, momental) theory of elasticity is considered to be the construction of applied theories of rods, plates and shells.

The main problem of the general theory of micropolar elastic thin rods, plates and shells is in the approximated but adequate reduction of the three-dimensional or two-dimensional problem of micropolar elasticity theory to a applied two-dimensional or one-dimensional problem. To attain this purpose, it is reasonable to use the qualitative results of the asymptotic analysis of the boundary problem of micropolar theory of elasticity in thin regions [1, 2]. In the present work accomplishing this task in a thin region of the shell, we have formulated sufficiently common assumptions (hypotheses) and as a result, depending on the values of sizeless physical constants, we have constructed models of micropolar elastic thin shells with independent displacements and rotations, constraint rotations and with "small shift rigidity" at which shear and related deformations are completely taken into account. Based on the constructed models concrete problems of applied nature on stress-deformed state of cylindrical shells in an axially symmetrical formulation are considered. The solution of these problems is brought to certain results. Based on the numerical analysis the most efficient properties of micropolar material are revealed.

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Theoretical Formulation and Numerical Solution of a Surface-Related Shell Model

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Abstract The main focus of this research is on the development of a surface-related shell model, i. e., the theoretical formulation and the numerical solution. The shell model is called surface-related because it uses one of the outer surfaces as a reference surface. The idea for the development of such a particular shell model is due to the SFB 528 (Collaborative Research Centre): “Textile Reinforcement for Structural Strengthening and Retrofitting” at Technische Universität Dresden. In the context of that SFB 528, technical textiles, that are used in fine grained concrete matrices for reinforcement, are developed and optimized. Therewith, a new innovative composite material, textile-reinforced concrete (TRC), was developed. This innovative construction material facilitates the design of very thin generally curved two-dimensional concrete members that can also be used for the strengthening of existing structures. The reason for the possible design of the very thin structure is based on the fact, that textile reinforcement is nearly noncorrosive compared to steel. TRC shows great strength and, as already pointed out, can be used for the strengthening of existing structures by thin strengthening layers that can be applied to the surface of existing structures only. The numerical simulation of the ultimate load-carrying capacity of such thin strengthening layers applied for strengthening is the main object of this research. Therefore, a mechanical model – a surface-related shell model – for the thin strengthening layers is developed [2].

Since shells are three-dimensional structures, i. e., material bodies in the sense of continuum mechanics, the field equations of continuum mechanics have to be the starting point. This set of partial differential equations with pertinent boundary conditions has to be solved. An efficient numerical solution of this problem becomes easier, if the problem is reformulated against a background of variational calculus. The discretization of the resulting variational formulation is, among others, the source of several locking phenomena.

The presented shell formulation uses linear shell kinematics with six displace-

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ment parameters. This low-order shell kinematics produces parasitical strains and stresses leading to very low convergence rates, i. e., locking. By locking, it is meant that finite element solutions, i. e., nodal displacements, vanish quickly to zero as certain parameters become very large. Due to the particular position of the reference surface an extension and/or adjustment of well-known techniques to prevent or reduce locking, like the assumed natural strain (ANS) method, cp. [3], and the enhanced assumed strain (EAS) method, cp. [4], is necessary.

Using these adapted methods, a reliable and efficient solid-shell element with massively reduced locking properties is obtained. This concept additionally comprises the utilization of unmodified three-dimensional constitutive relations by a minimal number of kinematical parameters, cp. [1]. With the aid of different non-linear examples, the reliability and the efficiency of the new solid-shell element is finally verified.

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An Algorithm for the Automatisations of Pseudo-reductions of PDE Systems Arising from the Uniform-approximation Technique

Patrick Schneider and Reinhold Kienzler

Abstract One way to develop theories for the elastic deformation of two or one-dimensional structures (like shells and beams, e.g.) under a given load is the uniform-approximation technique (see [1] for an introduction). This technique derives lower-dimensional theories from the general three-dimensional boundary value problem of linear elasticity by the use of series-expansions. It leads to a set of power series in one or two characteristic parameters, which are truncated after a given power, defining the order of the approximating theory. Finally, a so called pseudo-reduction of the resulting PDE system in the unknown displacement coefficients is performed, as the last step of the derivation of a consistent theory. The aim is to find a main differential equation system (at best a single PDE) in a few variables (at best only one) and a set of reduction differential equations, which express all other unknown variables in terms of the variables of the main differential equation system, so that the original PDE system is identically solved by inserting the reduction equations, if the main variables are a solution of the main differential equation system. To find a valid pseudo-reduction by inserting the PDEs of the original system into each other is a complicated and very time-consuming task for higher-order theories. Therefore an structured algorithm seeking all possibilities of valid pseudo-reductions (to a given number of PDEs in a given number of variables) is presented. The key idea is to reduce the problem to finding a solution of a linear equation system, by treating different products and derivatives of the same variables as independent variables. To this end, all necessary equations, which can be build from the original PDE system, have to be identified and added to the system a-priori.

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Characterization of Polymeric Interlayers in Laminated Glass for Photovoltaic Applications

Stefan-H. Schulze, Matthias Pander, Konstantin Naumenko, Anna Girchenko, and Holm Altenbach

Abstract Photovoltaic modules are designed as symmetrical laminate structures, consisting of a transparent front glass, a polymeric interlayer fixing the solar cells and a module back side. Especially in building integrated applications, these modules often are designed as laminated glass plates with a transparent silicate glass at the front and the back. During their estimated lifetime of at least 20-25 years, distributed snow loads, dynamic wind loads or punctual loading can occur, which can lead to critical deflections and, thus breakage of the cover glasses. Although these structures are tested in a distributed load test certification procedure before sold on the market, these tests only reflect the present state. The mechanical behavior of laminated glass is mainly depended on the compounding-effect that is mainly influenced by the soft polymeric interlayer. But polymers as organic materials can exhibit different aging effects which can strongly affect material and, thus structure properties. For this reason a practical and mathematical method to analyze laminated and weathered glass beams is necessary. In this work we present a mathematical model to describe the load-deflection behavior of laminated glass beams under three-point bending. On the basis of first order deformation shear theory a layerwise beam formulation is developed for calculating the maximum deflection of these beams. Results of this model are compared with bending tests on different laminated glass beams. It is shown, that these results allow a numerical calculation of polymer material parameters, which correspond to material parameters measured on the material itself.

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Influence of Coating Properties on Stability of Elastic Micropolar Rectangular Plate Subject to Biaxial Compression

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Abstract The present research is dedicated to the stability analysis of nonlinearly elastic plates of highly porous materials, such as metallic and polymeric foams. In majority of papers the behavior of foams is studied in the framework of a classic continuum model. But it seems more reasonable to use a model of micropolar medium or Cosserat continuum model for analysis of highly porous materials, because it allows describing their distinctive properties more accurately. Any particle of the Cosserat continuum has an additional inner degree of freedom - a microrotation. As a result of this a deformation in the couple-stress elasticity theory is described not by one, but by two strain tensors - Cauchy strain measure and bending strain tensor.

In this paper, in the framework of a general stability theory for three-dimensional bodies the buckling analysis has been carried out for a rectangular sandwich plate subject to biaxial compression. The sandwich plate consists of a porous core, covered by a hard and stiff shell. The behavior of a coating is investigated in the framework of a classic (non-polar) continuum model, while to describe the properties of a core the Cosserat continuum model is used. Using the linearization method in the vicinity of a basic state, the neutral equilibrium equations have been derived, which describe the perturbed state of a sandwich plate. By solving these equations numerically for some specific materials, the critical curves and corresponding buckling modes have been found, and the stability regions have been constructed in the plane of loading parameters (relative axial compressions). The influence of coating properties on the buckling of elastic micropolar rectangular plate subject to biaxial compression has been analyzed.

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Free Finite Rotations in Deformation Mechanics of Thin Bodies

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Abstract Invariant formulations of the generalized mathematical models of nonlinear deformation analysis of rod- and shell-like bodies are demonstrated in the lecture. The models are obtained from invariant formulation of the Cauchy's 3D-body nonlinear model with explicit introduction of finite local rotations.

The one-dimensional nonlinear model of a rod deformation is result of assumption on rigidly-rotating cross-sections. Three translational and three rotational degrees of section freedom generate the set of six primary unknown functions of the model. This model is equivalent the Cosserat's six-parametrical deformable curve if they stiffness parameters are coordinated.

The two-dimensional nonlinear model of a shell (and a plate) deformation is result of assumption on rigidly-rotating transversal fibres. Three translational and two rotational degrees of fibre freedom generate the set of five primary unknown functions of the model. Absence of one rotational degree of fibre freedom different this model from the Cosserat's six-parametrical deformable surface.

The generalized models, in addition to the axiomatic formulations, include constitutive relations of the real material and relations for computation of displacements, strains and stresses in volume of the real thin body.

Effectiveness of the generalized models is illustrated by numerical solutions of one-dimensional nonlinear problems of bending and buckling for arches, plates and shells.

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Theory of Shells in Application to Structural Analysis of Marine Propellers

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Abstract

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Shell-Models for Multi-Layered Carbon Nano-Particles

Melanie Todt, Franz G. Rammerstorfer, O. Paris, and Franz D. Fischer

Abstract Continuum mechanics modeling has shown to be an appropriate method to describe the mechanical behavior of different carbon nano-structures like single- and multi-walled carbon nanotubes, carbon onions, and carbon crystallites. Compared to atomistic modeling techniques the computational effort can be reduced significantly by proper continuum mechanics models, especially for multi-layered nano structures. In a continuum mechanics approach the atomic layers of such structures are modeled with thin shells characterized by an effective thickness, an effective elastic modulus, and a Poisson's ratio representing the in-plane and bending stiffness of the atomic layers. For multi-layered carbon nano structures the van der Waals interactions and covalent interlayer bonds have to be considered as well. The paper shows how continuum shell-models can be used to investigate the structural stability of carbon crystallites and carbon onions.

Carbon crystallites forming the nano structure of carbon fibers consist of a number of parallel graphene layers of finite extension. The compressive behavior of carbon fibers is closely related to the occurrence of crystallite buckling. With the current shell model of a single crystallite experimental observations presented in literature can be reproduced quite well. The shell model of the crystallite can, thus, serve as a basis for further investigations of the mechanical properties of carbon fibers.

In contrast to carbon crystallites carbon onions are doubly curved structures. The number of layers forming an onion seems to be limited. Current analyses, using an axi-symmetric shell-model of the onion, have shown that a structural instability can be a reason for this size limitation. One crucial point in investigating the stability of carbon onions using a continuum mechanics model is a correct description of the van der Waals interactions between the onion layers in a continuum mechanics

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framework. Taking into account the doubly curved nature of carbon onions in the formulation of the van der Waals model has an influence on the stress state in the individual layers and, hence, an influence on the occurrence of a structural instability of the onion.

Nonclassical Theories of Shells in Application to Soft Biological Tissues

Eva B. Voronkova, Svetlana M. Bauer, and Anders Eriksson

Abstract Shell-like structures are of very frequent occurrence in biological systems, either as body organs or their parts. Classical shell theories do not permit to take into account the specific structural features of soft biological tissue such as non-homogeneity and strong anisotropy. Two non-classical theories for orthotropic plates of moderate thicknesses are discussed in the present paper. Namely, the stress-strain state of an annular plate modelling the Lamina Cribrosa in the human eye is studied by means of 2D approximate refined theories: Rodionova-Titaev-Chernykh (RTC) and Paliy-Spiro (PS) [1, 2]. In these theories both deformations, rotation and bending of the fibers and their elongations in the direction of the thickness of the shell are taken into account. Results found with RTC and PS theories are compared with those obtained with FEM.

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Hybrid Structures Under Dynamic Loading

Dieter Weichert, Faten Ghazzai, and Russell Todres

Abstract In the present study, a circular aluminum plate with a bonded layer of unidirectional glass fibers is dynamically loaded and inelastically deformed in a shock wave tube. The load and displacement history of this hybrid structure are measured with microsecond resolution. Similar experiments are also carried out on a bare plate. Finite element simulations using Abaqus are also done for both cases using an appropriate shell theory. To model the metallic part of the composite, the Lemaitre-Chaboche viscoplastic material model is used with material parameters identified from separate tension tests. The results from experiments and simulations are compared to see how making the plate a composite modifies its damping and energy absorption characteristics and overall stress state.

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On the Determination of Edge Reinforcement Properties for Optimum Lightweight Design of Composite Stiffeners

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Abstract In this work the determination of the properties of an edge reinforcement of a composite plate stiffener is treated. A minimum stiffness criterion for the edge reinforcement on the basis of a closed-form buckling analysis of a composite plate with edge reinforcement and elastic clamping is given. The minimum stiffness criterion is given in explicit form and in a fully dimensionless representation. A composite stiffener designed by this criterion will exhibit a local buckling mode of the web, rather than a global simultaneous buckling of both the web and the edge reinforcement. It is shown that there is a limit value of the required bending stiffness of the edge reinforcement, which is approached with higher aspect ratios, that is sufficient to ensure local buckling for arbitrarily long stiffeners. In an example the criterion is applied to the dimensioning of a stiffener design.

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Influence of Initial Geometrical Imperfections of Random Size and Location on a Silos Bearing Capacity under Negative Pressure and Wind Loading

Karol Winkelmann, Magdalena Gołota, Jarosław Górski, and Tomasz Mikulski

Abstract Silos as examples of cylindrical shell structures with geometric imperfections are considered. The silos geometric imperfections can apparently affect their loading capacity so they should be necessarily included in computations and in the design process. The imperfection dimensions are properly limited by EuroCode 3 design rules [1]. For the random mesh of imperfections alternative formulations are applied both correlated and uncorrelated random fields are used [2]. Theoretical assumptions of the random field formulas of the geometrical discrepancies are provided. Wind loading is also considered in different formulations. The external loading parameters are assumed using a set of measured data. The Gumbell distribution is applied for the random field descriptions.

It has been shown, that under the main loading, acting on the silos negative pressure and wind pressure for example a single dent or a randomly generated mesh of imperfections shows radical differences in comparison with analogous results for the construction with a perfect geometry. It is also shown, that the number and location of the imperfections also differentiates the results obtained.

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Advantages and Disadvantages of Different Methods of Modeling of Thin Plates Made of Functionally Graded Materials

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Abstract The subject of our consideration are thin plates made of a two-phase functionally graded composite. These plates have a special structure: periodic in one direction, while in the second, perpendicular direction, their averaged properties change as a slowly varying function. The objective of the paper is comparison of accuracy and computational effort of various methods of modeling of thin plates with the proposed structure.

There will be analyzed three ways:

1. The finite element method,
2. Engineering and intuitive method that involves a simple averaging in the direction in which the plate has a periodic properties and bringing problem to the one-dimensional, and then solve it using the finite difference method,
3. The tolerant averaging method, which takes account a microscale effect.

The numerical examples will be restricted to the problem of free vibrations. The paper presents two types of plates with the given structure: band plate and annular plate. In its conclusions, summarizes the results obtained using different methods in terms of their accuracy and computational effort.

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Recent Improvements in Hu-Washizu Shell Elements

Krzysztof Wiśniewski

Abstract Mixed and mixed/enhanced 4-node elements based on the Hu-Washizu (HW) functional are developed for Green strain and non-linear constitutive laws modified by the zero-normal-stress condition. They are used as a membrane part of either the shell elements using the rotational degrees-of-freedom (including the drilling rotation) or the 'solid-shell' elements. The key features of our approach, see [1-5], are as follows.

1. The representations of stress and strain are assumed in skew coordinates. In consequence, the homogenous equilibrium equations and the compatibility condition are satisfied point-wise for an arbitrary shape of element, see [3,4]. The discrete LBB condition is satisfied.
2. The Hu-Washizu functional is used in either a mixed or mixed/enhanced form and for each the optimal assumed representations are selected. In particular, the performance of the elements with reduced number of parameters will be illustrated, see [2].

The developed elements exhibit excellent accuracy and robustness to shape distortions for coarse meshes, and perform very well for finite strains. The issue of an increased radius of convergence of the Hu-Washizu shell elements will be addressed.

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On the Large Deformations of Nonlinear Isotropic Elastic Shells Without Surface Loads

Leonid M. Zubov

Abstract A two-dimensional nonlinear model of elastic shell is considered in the paper. The model assumes that the shell is a deformable surface with kinematically independent fields of translations and rotations. Within this theory families of finite deformations are found satisfying equilibrium equations for any isotropic shell without surface loads. These solutions we call universal ones. Obtained universal solutions consists six families of deformations that are characterized by certain field of surface displacements. Each of these families consists of several subfamilies that differ in rotation fields. One of the results of the paper is that a closed spherical isotropic shell without external loads has four different equilibrium states, and the shell remains spherical in each of these equilibrium states.

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Important Dates & Publication

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- Registration opens on: 01/01/2011
- Registration deadline: 01/03/2011
- Camera-ready paper submission deadline: 01/05/2011
- On-site registration: 21/08/2011
- Conference dates: 22-26/08/2011

Publication

The conference papers accepted after review will be published by Springer in the Series "**Advanced Structured Materials**"

<http://www.springer.com/series/8611>

and will be available at the Conference. The manuscript should be prepared with the textprocessing system **LATEX** using the template `svmult.zip`. The template can be downloaded from

http://tm.iw.uni-halle.de/euromech-2011/euromech-527_proceedings/

Fees

Conference and other Fees

Euro	
200	Regular participant (EUROMECH member)
224	Regular participant (non EUROMECH member)
10	City tour Wittenberg
40	Excursion
50	Banquet

The regular fee includes: welcome reception, proceedings, and coffee breaks.

City tour, excursion, and banquet should be booked for each person (participant and accompanying person) separately.

The conference fee should be paid before **1 May 2011** by bank transfer:

Überweisung innerhalb Deutschlands:

Kontonr. 4000381380

BLZ 81053272

Stadtsparkasse Magdeburg

Empfänger: Prof. Dr. Holm Altenbach

Verwendungszweck: EUROMECH 527 und Name des Teilnehmers

Transfer from abroad:

IBAN: DE17 8105 3272 4000 3813 80

BIC-/SWIFT-Code: NOLADE21MDG

Stadtsparkasse Magdeburg

Account holder: Prof. Dr. Holm Altenbach

Please indicate EUROMECH 527 and the name of the participant

Venue

The conference venue is the LEUCOREA - the oldest part of the Martin-Luther-University Halle-Wittenberg. The LEUCOREA was founded in 1502 and renovated in the 90's of the last century. Today there are a big auditorium with all necessary facilities, small conference rooms, several research institutes, and last but not least accommodation possibilities (38 single rooms and 6 double rooms). The LEUCOREA



Fig. 1 Leucorea

is located downtown in the Lutherstadt Wittenberg. Nearby the LEUCOREA there are several hotels and many restaurants, pubs, etc. For further information please visit the homepage of Wittenberg
<http://www.wittenberg.de/>

How to go to Wittenberg

You can go to the Lutherstadt Wittenberg by train (ICE/IC from Berlin, Leipzig or Halle) or by car (highway A9 from Berlin to Munich). For detailed information of your train connection please check the homepage of "Deutsche Bahn" (German Railway), by following this site <http://reiseauskunft.bahn.de> The nearest airports are located in Berlin-Tegel (TXL) or Berlin-Schnefeld (SFX) and between Leipzig and Halle (LEJ).

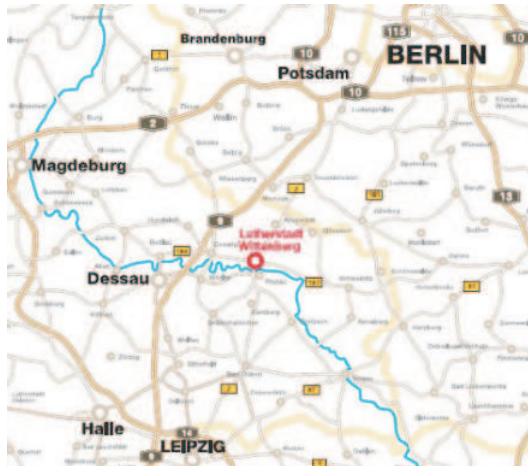


Fig. 1 Map

