Numerical Simulation of Elongated Fibres in Horizontal Channel Flow

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Introduction

- In engineering predictions of dispersed two-phase flow spherical shape for particles is the general assumption.
- Most practical situations particles are irregular or have certain shape (e.g., granulates or fibres)
- Dynamics of non-spherical particles are substantially different from that of spherical (i.e., pitching and rotational torques)
- Analytical expressions of forces & moments are known for non-spherical regular particles in Stokes regime ($\text{Re}_p << 1$)
- For moderate $\text{Re}_p$, not much information available
- In such case, fully resolved DNS can be used to extract flow coefficients depending on $\text{Re}_p$ and particle shape (e.g., Hölzer & Sommerfeld, 2009; Vakil & Green, 2009; Zastawny et al., 2012)
- Final goal: develop an engineering tool to predict dispersed two-phase flows laden with non-spherical particles
Numerical approximation

- System of interest: behaviour of non-spherical particles immersed in turbulent channel flow
- System described by the Euler-Lagrange approach
- Particles considered as point masses with dynamics given by linear and angular (orientation) momentum equations
- Fluid field computed by RANS (Reynolds Stress Model), modified by the presence of particles (two-way coupling)
- Non-spherical particles motion due to drag and lift forces, whose coefficients were previously obtained by DNS, as well as pitching and rotational torques.
- Here, elongated fibres at intermediate Reynolds numbers are considered. Expressions for the interaction with solid walls have been developed.
- Outputs: particle mean velocity, fluctuating velocity components (stream-wise and vertical) and concentration profiles in the channel.
Non-spherical particles governing equations

\[ \vec{x}' = A \cdot \vec{x}'' \]

Rotation matrix \( A \) written in terms of Euler parameters

\[
A = \begin{bmatrix}
1 - 2(\varepsilon_2^2 + \varepsilon_3^2) & 2(\varepsilon_1 \varepsilon_2 + \varepsilon_3 \eta) & 2(\varepsilon_1 \varepsilon_3 - \varepsilon_2 \eta) \\
2(\varepsilon_1 \varepsilon_2 - \varepsilon_3 \eta) & 1 - 2(\varepsilon_1^2 + \varepsilon_3^2) & 2(\varepsilon_3 \varepsilon_2 + \varepsilon_1 \eta) \\
2(\varepsilon_1 \varepsilon_3 + \varepsilon_2 \eta) & 2(\varepsilon_3 \varepsilon_2 - \varepsilon_1 \eta) & 1 - 2(\varepsilon_2^2 + \varepsilon_1^2)
\end{bmatrix}
\]

Time evolution of Euler parameters

\[
\begin{bmatrix}
\frac{d\varepsilon_1}{dt} \\
\frac{d\varepsilon_2}{dt} \\
\frac{d\varepsilon_3}{dt} \\
\frac{d\eta}{dt}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\eta \omega_x' - \varepsilon_3 \omega_y' + \varepsilon_2 \omega_z' \\
\varepsilon_3 \omega_x' + \eta \omega_y' - \varepsilon_1 \omega_z' \\
-\varepsilon_2 \omega_x' + \varepsilon_1 \omega_y' + \eta \omega_z' \\
-\varepsilon_1 \omega_x' - \varepsilon_2 \omega_y' - \varepsilon_3 \omega_z'
\end{bmatrix}
\]
Non-spherical particles governing equations

Translational motion (inertial frame)

\[ m_p \frac{du_p}{dt} = \vec{F} \]

Rotational motion (particle frame)

\[ I_x' \frac{d\omega_{x'}}{dt} - \omega_y' \omega_z' (I_y' - I_z') = T_x' \]
\[ I_y' \frac{d\omega_{y'}}{dt} - \omega_x' \omega_z' (I_z' - I_x') = T_y' \]
\[ I_z' \frac{d\omega_{z'}}{dt} - \omega_y' \omega_x' (I_x' - I_y') = T_z' \]

Forces: drag and lift

Torques: pitching and rotational
Flow coefficients cylindrical fibres

Cylinders Vakil & Green, C&F (2009)

Correlations flow coefficients obtained by DNS depending on particle Reynolds numbers, orientation and aspect ratio (AR)

\[ C_{D,cyl} = \frac{F_D}{\frac{1}{2} \rho \tilde{u}^2 LD} \]
\[ C_{L,cyl} = \frac{F_L}{\frac{1}{2} \rho \tilde{u}^2 LD} \]

Torque coefficients follow the approach of Yin et al., CES (2003)
Non-spherical particle – wall interaction

$C : (x, y, z) \rightarrow (r_c, \beta, \gamma)$

$0 \leq \gamma \leq 2\pi; \ 0 \leq \beta \leq \pi$

- $\beta$, $\gamma$ and $r_c$ depend on shape and orientation of particle
- $\beta$, $\gamma$ and $r_c$ are determined using real values of $\theta$, $\phi$ and $\psi$ before wall collision and are determined using an analytic model
- A orientation matrix $A$ is necessary defined to get the orientation the particle before wall collision
Non-spherical particle – wall interaction

Hard particle approximation

\[ m(v^{(2)} - v^{(0)}) = J \]
\[ l(\omega^{(2)} - \omega^{(0)}) = -\mathbf{r} \times \mathbf{J} \]

Crowe et al. (2011)

I. Particle stops sliding in compression period
II. Particle stops sliding in recovery period
III. Particle slides throughout compression and recovery period

Additional Assumptions:
Collision is modeled by the coefficient of restitution and friction is modeled by Coulomb’s law
Non-spherical particle – wall interaction

Case I

Linear velocities

\[ u^{(2)} = \omega_y^{(2)} r_c \cos \beta - \omega_z^{(2)} r_c \sin \beta \sin \gamma \]
\[ v^{(2)} = \omega_z^{(2)} r_c \sin \beta \cos \gamma - \omega_x^{(2)} r_c \cos \beta \]
\[ w^{(2)} = w^{(0)} + (1 + e) J_z^{(1)} / m_p \]

Angular velocities

\[ \omega_x^{(2)} = \frac{l_x \omega_x^{(0)} - m_p v^{(0)} r_c \cos \beta - (1 + e) r_c \sin \beta \sin \gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin \beta \cos \beta \cos \gamma}{l_x + m_p r_c^2 \cos^2 \beta} \]
\[ \omega_y^{(2)} = \frac{l_y \omega_y^{(0)} + m_p u^{(0)} r_c \cos \beta + (1 + e) r_c \sin \beta \cos \gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin \beta \cos \beta \sin \gamma}{l_y + m_p r_c^2 \cos^2 \beta} \]
\[ \omega_z^{(2)} = \frac{l_z \omega_z^{(0)} + m_p v^{(0)} r_c \sin \beta \cos \gamma - m_p u^{(0)} r_c \sin \beta \sin \gamma + m_p \omega_y^{(2)} r_c^2 \sin \beta \cos \beta \cos \gamma - m_p \omega_x^{(2)} r_c^2 \sin \beta \cos \beta \cos \gamma}{l_z + m_p r_c^2 \sin^2 \beta} \]

Linear impulse normal to the wall

\[ J_z^{(1)} = m_p (\omega_x^{(1)} r_c \sin \beta \sin \gamma - \omega_y^{(1)} r_c \sin \beta \cos \gamma - w_0) \]
\[ \omega_y^{(1)} = \frac{m_p l_y h_2 \omega_y^{(0)} + m_p l_x h_2 \omega_x^{(0)} + m_p l_z l_3 \omega_z^{(0)} + m_p r_c \cos \beta u_0 l_1}{m_p l_y r_c^2 \sin^2 \beta \sin^2 \gamma (l_x + l_z + m_p r_c^2) + m_p l_x r_c^2 \cos^2 \beta \sin^2 \gamma (l_y + l_z + m_p r_c^2) + m_p l_y r_c^2 \cos^2 \beta (l_x + l_y + m_p r_c^2) + l_x l_y l_z} \]
\[ \omega_x^{(1)} = \frac{m_p l_y h_1 \omega_x^{(0)} + m_p l_y h_3 \omega_y^{(0)} + m_p l_z l_3 \omega_z^{(0)} + m_p r_c \cos \beta u_0 l_4}{m_p l_y r_c^2 \sin^2 \beta \sin^2 \gamma (l_x + l_z + m_p r_c^2) + m_p l_x r_c^2 \cos^2 \beta \sin^2 \gamma (l_y + l_z + m_p r_c^2) + m_p l_y r_c^2 \cos^2 \beta (l_x + l_y + m_p r_c^2) + l_x l_y l_z} \]
Non-spherical particle – wall interaction

Case I

Extra relations

\[ l_1^2 = m_p r_c^2 \sin^2 \beta \cos^2 \gamma l_x + m_p r_c^2 \sin^2 \beta \sin^2 \gamma l_z + l_x l_z \]
\[ l_2^2 = m_p r_c^2 \sin \beta \sin \gamma \cos \beta l_z - m_p r_c^2 \sin \beta \sin \gamma \cos \beta l_x \]
\[ l_3^2 = m_p r_c^2 \sin^2 \beta \cos^2 \gamma l_x + m_p r_c^2 \sin^2 \beta \sin^2 \gamma l_z + m_p r_c^2 \cos^2 \beta l_z + l_y l_z \]
\[ l_4^2 = m_p r_c^2 \sin \beta \cos \gamma \cos \beta l_z - m_p r_c^2 \sin \beta \cos \gamma \cos \beta l_x \]
\[ l_5^2 = m_p r_c^2 \sin^2 \beta \cos^2 \gamma l_z + m_p r_c^2 \sin^2 \beta \sin^2 \gamma l_y + m_p r_c^2 \cos^2 \beta l_z + l_y l_z \]
\[ l_6^2 = m_p r_c^2 \sin^2 \beta \cos^2 \gamma l_y + m_p r_c^2 \sin^2 \beta \sin^2 \gamma l_z + m_p r_c^2 \cos^2 \beta l_z + l_x l_z \]
\[ l_{11} = m_p r_c^4 \sin^2 \beta \cos^2 \gamma + r_c^2 \cos^2 \beta l_z + r_c^2 \sin^2 \beta \sin^2 \gamma l_y + r_c^2 \sin^2 \beta \cos^2 \gamma (l_y + l_z) + l_y l_z / m_p \]
\[ l_{22} = m_p r_c^4 \sin^2 \beta \sin^2 \gamma + r_c^2 \cos^2 \beta l_z + r_c^2 \sin^2 \beta \cos^2 \gamma l_x + r_c^2 \sin^2 \beta \cos^2 \gamma (l_x + l_z) + l_x l_z / m_p \]
\[ l_{12} = l_{23} = m_p r_c^2 \sin \beta \sin \gamma \cos \beta + r_c^2 \sin^2 \beta \sin \gamma \cos \beta l_z \]
\[ l_{31} = m_p r_c^2 \sin \beta \sin \gamma \cos \beta + r_c^2 \sin^2 \beta \sin \gamma \cos \beta l_x \]
\[ l_{32} = m_p r_c^2 \sin \beta \cos \gamma \cos \beta + r_c^2 \sin^2 \beta \cos \gamma \cos \beta l_x \]
Non-spherical particle - wall interaction

Case II

Linear velocities

\[
\begin{align*}
u^{(2)} &= \omega^{(2)}_y r_c \cos \beta - \omega^{(2)}_z r_c \sin \beta \sin \gamma \\
v^{(2)} &= \omega^{(2)}_z r_c \sin \beta \cos \gamma - \omega^{(2)}_x r_c \cos \beta \\
w^{(2)} &= w^{(0)} + (1 + e) J^{(1)}_z / m_p
\end{align*}
\]

Angular velocities

\[
\begin{align*}
\omega^{(2)}_x &= \frac{l_x \omega^{(0)}_x - m_p v^{(0)} r_c \cos \beta - (1 + e) r_c \sin \beta \sin \gamma J^{(1)}_z + m_p \omega^{(2)}_z r_c^2 \sin \beta \cos \beta \cos \gamma}{l_x + m_p r_c^2 \cos^2 \beta} \\
\omega^{(2)}_y &= \frac{l_y \omega^{(0)}_y + m_p u^{(0)} r_c \cos \beta + (1 + e) r_c \sin \beta \cos \gamma J^{(1)}_z + m_p \omega^{(2)}_z r_c^2 \sin \beta \cos \beta \sin \gamma}{l_y + m_p r_c^2 \cos^2 \beta} \\
\omega^{(2)}_z &= \frac{l_z \omega^{(0)}_z + m_p v^{(0)} r_c \sin \beta \cos \gamma - m_p u^{(0)} r_c \sin \beta \sin \gamma + m_p \omega^{(2)}_y r_c^2 \sin \beta \cos \beta \sin \gamma - m_p \omega^{(2)}_x r_c^2 \sin \beta \cos \beta \cos \gamma}{l_z + m_p r_c^2 \sin^2 \beta}
\end{align*}
\]

Linear impulse normal to the wall

\[
f^{(1)}_z = \frac{m_p l_x l_y (\omega^{(0)}_x r_c \sin \beta \sin \gamma - \omega^{(0)}_y r_c \sin \beta \cos \gamma - w^{(0)})}{l_x l_y + m_p r_c^2 [l_x (\varepsilon_x \mu \sin \beta \cos \beta \cos \gamma + \sin^2 \beta \cos^2 \gamma) + l_y (\varepsilon_y \mu \sin \beta \cos \beta \sin \gamma + \sin^2 \beta \sin^2 \gamma)]}
\]
Non-spherical particle - wall interaction

Case III

Linear velocities

\[ u^{(2)} = u^{(0)} - \varepsilon_x \mu (1 + e) \frac{J_z^{(1)}}{m_p} \]
\[ v^{(2)} = v^{(0)} - \varepsilon_y \mu (1 + e) \frac{J_z^{(1)}}{m_p} \]
\[ w^{(2)} = w^{(0)} + (1 + e) \frac{J_z^{(1)}}{m_p} \]

Angular velocities

\[ \omega_x^{(2)} = \omega_x^{(0)} - (1 + e) [\varepsilon_y \mu \cos \beta + \sin \beta \sin \gamma] \frac{J_z^{(1)} r_c}{l_x} \]
\[ \omega_y^{(2)} = \omega_y^{(0)} + (1 + e) [\varepsilon_x \mu \cos \beta + \sin \beta \cos \gamma] \frac{J_z^{(1)} r_c}{l_y} \]
\[ \omega_z^{(2)} = \omega_z^{(0)} + (1 + e) [\varepsilon_y \mu \cos \gamma - \varepsilon_x \mu \sin \gamma] \sin \beta \frac{J_z^{(1)} r_c}{l_z} \]

Linear impulse normal to the wall

\[ J_z^{(1)} = \frac{m_p l_x l_y (\omega_x^{(0)} r_c \sin \beta \sin \gamma - \omega_y^{(0)} r_c \sin \beta \cos \gamma - w^{(0)})}{l_x l_y + m_p r_c^2 [l_x (\varepsilon_x \mu \sin \beta \cos \beta \cos \gamma + \sin^2 \beta \cos^2 \gamma) + l_y (\varepsilon_y \mu \sin \beta \cos \beta \sin \gamma + \sin^2 \beta \sin^2 \gamma)]} \]
Non-spherical particle – wall interaction

Conditions for occurrence of the cases

Case I & II

\[
\frac{w^{(0)}}{u^{(0)} - \omega_y^{(0)} r_c \cos \beta + \omega_z^{(0)} r_c \sin \beta \sin \gamma} < -\frac{1}{(1 + m_p r_c^2 \cos^2 \beta / l_y) \mu (e + 1)}
\]

Case III

\[
-\frac{1}{(1 + m_p r_c^2 \cos^2 \beta / l_y) \mu (e + 1)} < \frac{w^{(0)}}{u^{(0)} - \omega_y^{(0)} r_c \cos \beta + \omega_z^{(0)} r_c \sin \beta \sin \gamma} < 0
\]

Above equations reduce in the 2D case to those of Sommerfeld (2002) and in the case of spherical particles to those of Crowe, Sommerfeld, Tsuji (1998).
Non-spherical particle – wall interaction

Case IV, ideal collision ($e = 1, \mu = 0$)

Linear velocities

\[
\begin{align*}
    u^{(2)} &= u^{(0)} \\
    v^{(2)} &= v^{(0)} \\
    w^{(2)} &= w^{(0)} + \frac{2J_z^{(1)}}{m_p}
\end{align*}
\]

Angular velocities

\[
\begin{align*}
    \omega_x^{(2)} &= \omega_x^{(0)} - 2 \sin \beta \sin \gamma \frac{J_z^{(1)}r_c}{l_x} \\
    \omega_y^{(2)} &= \omega_y^{(0)} + 2 \sin \beta \cos \gamma \frac{J_z^{(1)}r_c}{l_y} \\
    \omega_z^{(2)} &= \omega_z^{(0)}
\end{align*}
\]

Linear impulse normal to the wall

\[
J_z^{(1)} = \frac{m_pl_xl_y(\omega_x^{(0)}r_c \sin \beta \sin \gamma - \omega_y^{(0)}r_c \sin \beta \cos \gamma - w^{(0)})}{l_xl_y + m_pr_c^2 \sin^2 \beta(l_x \cos^2 \gamma + l_y \sin^2 \gamma)}
\]
Non-spherical particle – wall interaction

Simplified 2D equation cylinders (ideal case)

\[
\frac{w^{(2)}}{w^{(0)}} = 1 - 2 \left( \frac{1 + r_c \omega_y^{(0)} \sin \beta / w^{(0)}}{1 + (r_c / k_y)^2 \sin^2 \beta} \right)
\]
Fibre – wall interaction visualisation
Experimental rig

Channel Flow Configuration

Experimental studies by Kussin (2004)

**Horizontal Channel:**
- Length: 6 m
- Width: 350 mm
- Height: 35 mm
- $U_{av} = 20$ m/s
- Measuring section: 5.8 m

Spherical glass beads, $\rho_p = 2450$ kg/m$^3$
**Numerical set-up**

**Horizontal Channel:**
6 m x 350 mm x 35 mm
\( U_{av} = 20 \text{ m/s} \)
Measuring section: 5.8 m

- \( \rho = 1.25 \text{ kg/m}^3 \)
- \( \mu = 1.8 \cdot 10^{-5} \text{ N} \cdot \text{s/m}^2 \)
- \( \rho_p = 2450 \text{ kg/m}^3 \)
- \( D_p = 130 \mu\text{m} \)
- \( \eta = 0.1 \)

**Degree of roughness**

<table>
<thead>
<tr>
<th>Degree of roughness</th>
<th>Mean roughness in stream-wise direction [( \mu\text{m} )]</th>
<th>Mean roughness in lateral direction [( \mu\text{m} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0 (LR)</td>
<td>2.32</td>
<td>2.09</td>
</tr>
<tr>
<td>R2 (HR)</td>
<td>6.83</td>
<td>6.89</td>
</tr>
</tbody>
</table>

**Wall roughness (stochastic model)**

- Spheres
- Fibres (aspect ratio 13)

**Tracking of 240,000 parcels**
Results cylindrical particles
Results cylindrical particles
Results cylindrical particles
Results cylindrical particles
Conclusions

- The Euler/Lagrange approach has been used to investigate the transport of spheres and elongated fibres in a turbulent channel flow.
- Equations for the interaction of general non-spherical particles with walls have been presented in the context of “hard particles.”
- Depending on angle of impact and rotational velocity, one or two collisions with the wall can be observed.
- As expected, fibres follow better than spheres the fluid flow, as they tend to be aligned perpendicular to the relative velocity, maximizing the drag.
- From the concentration profiles, fibres show more gravitational settling than spheres of the same size.
- Streamwise rms velocity of fibres is more asymmetric than that of spheres.
- Vertical rms velocity is remarkably lower in the case of elongated cylinders than for spherical particles, as happens with other type of irregular shapes.