An Alternative Approach to a Child-related Pension

by Sven Tagge
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Abstract

In this article, the effects of institutional conditions on fertility behavior are analyzed. In the broad literature it is shown that the shape of the pension system can negatively influence fertility decisions. Taking this as a starting point we are looking for an alternative pension scheme that does not generate that deficiency. We find that the introduction of a child-related pension system increases the optimal number of children in households. But in a neoclassical growth environment, the introduction of the new system is associated with a smaller maximum utility level for the individuals compared with fully-funded systems.

Key words: Endogenous fertility, Child-related pension system, Governmental sponsorship
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1 Introduction

Institutional conditions within a society influence the human decisions of the members of that society. The provision of publicly financed childcare, of education or of alternative means of old-age security are examples of institutional arrangements outside the family that impinge on the costs and benefits of the offspring. A broadly discussed field of research is the effects of the shape of the pension scheme on fertility incentives. This is well known under the synonym of the Old Age Security Hypothesis. As described by Cigno (1992, pp. 176-178), according to this hypothesis, the concern of individuals to provide for their retirement period is an important driving force behind the demand for children. Therefore, the appearance of alternative modes of old-age security leads to a reduction of the relevance of offspring as an instrument of old-age support.

There is a range of articles that indicate a negative effect of pay-as-you-go (PAYG) pension systems on the fertility incentives of households. For instance, in the model of Zhang/Zhang (1998, pp. 1231, 1234) the rise in the contribution rate to the PAYG system leads to declining fertility, because social security replaces children as a means of old-age support. The reason is that in earnings-related PAYG systems the entitlements to pension payments and the ability to provide pension payments become separate. As described by Steurer (2002, pp. 9-10), in earnings-related PAYG systems the pension entitlements are directly proportional to the lifetime earnings of the individual and are totally independent of the number of offspring. But average fertility determines the level of pension payments. Thus, for an individual it is not necessary to have children of one’s own in order to receive old-age support from the PAYG system, since the individuals ignore the marginal effect of their own fertility decision on the average fertility. Furthermore, as stated by Cigno (1992, p. 178) for instance, in a PAYG system the contributions made by the additional child benefit later retirees in general and not only the parents who bear the corresponding childrearing costs. In this way, every child is associated with a positive externality for the community.

With the capital market as an alternative source of old-age security, individuals have to decide whether to invest further in children or to invest in savings. In the model of Cigno/Rosati (1992, p. 320-322), this decision problem is represented by the question of staying in the family-based transfer system or leaving the club and accumulating savings for retirement. They argue that for totally selfish parents the family-based transfer system immediately breaks down when the ”domestic” interest rate is not sufficiently higher than the interest rate on the capital market.
this case, the parents would substitute savings for children as old-age insurance. Both elucidated alternatives have in common that children are not integrated explicitly in the entitlement structure. Thus, the expectable pension payments are independent of the recipients’ own number of children. The above-mentioned can be subsumed with the following statement from Robinson (1997, p. 66) on the existence of service substitutes for children: "And, when alternative sources of labour, leisure-time amusement, and of future economic security, are available the demand for child-services becomes a function of the opportunities and prices in these other markets. On purely economic grounds, there is no reason why a well-endowed couple will ever 'need' children to provide any service for them.”.

Empirical support of the Old Age Security Hypothesis comes from a wide range of articles: for example, Cigno/Rosati (1992) tested the accessibility to the capital market and the availability of a social security system. They found support for the hypothesis that wider access to the capital market as well as wider social security coverage reduces fertility. Cigno/Casolaro/Rosati (2000) and Ehrlich/Zhong (1998) also found empirical evidence that social security coverage through a PAYG system has a negative effect on the fertility incentives of households.

There are several approaches to re-establishing the fertility incentives of households. A starting point is the incorporation of the effort associated with childrearing into the old-age security system: for example, the suggestion of a hybrid pension system by Sinn (1997, pp. 10-11, 14-17), where funding is a complement to the PAYG system. Every person has to contribute to the funded and to the PAYG system. But those with children get a rebate for their parenting effort, measured by the external effect every child generates for the society. He argues that the internalization of this external effect would have a strong impact on the fertility decisions of households. But this is far from clear, because childrearing must be seen in comparison with its alternatives for retirement security. That is, as long as children are poor investments compared with savings, individuals could decide on additional funding, despite an anticipated rebate for children.

Another proposal comes from Fenge/Meier (2003, pp. 5-8), who incorporate a child factor into the PAYG system to make the pensions at least partly dependent on the number of an individual’s offspring. They show that introducing the child factor always raises fertility, because of the higher marginal return from children. Moreover, savings would decrease if children and savings were substitutes for each other. They also argue, that incorporating the external effect of a child into the PAYG system could be the worse alternative compared with child allowances. This would be the case if the internal rate of return in the PAYG system were smaller than the
market interest rate.\footnote{Fenge/Meier (2003, p. 17)} Finally, the returns from the PAYG system for the individuals remain partly dependent on the average fertility of this particular society, because the optimal child factor is always smaller than one. Let us move on from the usual PAYG framework and the matter of financing such a pension system and turn to the search for an alternative old-age security system that has the property of not depressing fertility. Let us examine the idea of binding the benefits of pensions to the effort of childrearing the own children. But we do not rely on a family governance structure, as in the article of Cigno/Rosati (1992). We create a state-governed entitlement structure that enforces the entitlements of parents and guarantees comparable returns from investment in children and in capital savings. Additional funding is often seen as the alternative to pensions coming from offspring to maintain the pension system. Accordingly, we compare our child-related pension system only with the situation where capital savings are the sole source of old-age security and put aside the direct comparison to the usual PAYG system. This paper is organized as follows: in subsection 2.1 we give an introduction to the above mentioned alternative pension system. In subsection 2.2 we analyze the optimal behavior of households and look at the effects on fertility arising from the introduction of such a pension scheme. We can observe that raising the fraction of accountable childrearing costs leads to a higher optimal fertility rate. After that, in subsection 2.3 we introduce a neoclassical-shaped production sector and analyze the equilibrium of that economy in subsection 2.4. Furthermore, we provide a comparative statics analysis in section 3 and compare the welfare outcome of child-related and of capital pensions in section 4. We ascertain that the introduction of the suggested child-related pension system can lead to overall smaller maximum utility for the individuals, compared with funding alone. Finally, in section 5 we elaborate some numerical simulations to underpin our analytical results. In section 6 we conclude.

## 2 A model of sponsorship

We consider three overlapping generations of individuals of the same sex. The total population in period $t$ is composed of children (1), workforce (2) and retirees (3): $N_t = N_t^1 + N_t^2 + N_t^3$. Every individual lives three periods and dies at the end of the retirement period. Consumption by children, the workforce and retirees is $C^1, C^2$
and $C^3$ respectively. Actually, an individual wants to maximize his lifetime utility over three periods. But it is relatively unrealistic to assume that children are able to decide about their own consumption in childhood. Because of that, the parents fix the consumption of their children. For the sake of simplicity, one can assume that the children themselves do not get any utility from their consumption in childhood. Individuals begin to maximize their lifetime utility from the start of adulthood and achieve a direct utility from the number of children they have.

2.1 Pension entitlements and the costs of childrearing

We present an alternative approach of a pension system where the state plays the role of a sponsor. This pension scheme is characterized by three principles:

1. The pension entitlements of individuals are directly connected to the childrearing effort of these individuals.
2. The special position of the state is used to vouch for and to enforce the pension entitlements of individuals.
3. The shape of this pension system is also characterized by the so-called Hobbes-Rousseau social contract.\(^3\)

First, childrearing is associated with costs for the parents and a certain fraction of these costs can be credited for old-age security. Second, every household possesses its own pension account in which the corresponding childrearing costs are registered, such that the repayment (pension) by one’s own children is exactly calculable.\(^4\) To account for this childrearing effort, the parents have to buy credit notes from the state in period $t$, which reflect the associated fostering costs for their children. Third, in return the state spends the incoming funds immediately as transfers to the corresponding children of those parents. For example, one could imagine vouchers for day care, schooling or study fees. Fourth, in the following period $t+1$ the children have to pay back the value of these credit notes plus an interest as pensions to their own parents. This is exactly consistent with the credited part of their childrearing costs.

Through the aforementioned pension scheme, a certain fraction of the childrearing costs is accountable for the pensions of the corresponding parents. In this way, the

\(^3\)See Samuelson (1958, pp. 479-480)
\(^4\)This is like the pension accounts of individuals in a PAYG system, for instance the German PAYG system, except that the childrearing costs instead of the earnings-related contributions will be registered.
working individuals bear a part of their parenting costs just for a certain period within their lifetime, because they obtain backward payments (pensions) from their children later, according to the childrearing costs incurred. Because the offspring will bear the repayment themselves, in the end every individual pays at least a part of his own childrearing costs. In Appendix A.1, this problem is illustrated in the form of accounts for members of a single family and for the state.

In more detail: The total offspring costs of a worker (which is the consumption of all children \( C_1^t \)) are composed by
\[
C_1^t = (1 - b)hn_t^d w_t + bhn_t^d w_t = hn_t^d w_t,
\]
whereby \( 0 < h < 1 \) and \( d \geq 1 \) are constants and \( n_t \) is the number of children per worker in period \( t \). Because there are some facts that speak for a progressive and some other facts that speak for a diminishing course of childrearing costs, we choose the middle course of linear child costs: \( d = 1 \). In addition, a progressive course of the child cost function would not alter the results qualitatively. In the case of linear child costs, \( h \) represents the fraction of wage income which is necessary to rear one child. The parameter \( 0 < b < 1 \) reflects the fraction of the childrearing costs that can be credited for old-age security. The part of costs \( (1 - b)hn_t^d w_t \) remains with the parents. As truncated above, the other part of the childrearing costs can be financed by credit notes issued by the government. That is, the state issues credit notes \( T_t \) to the price \( 0 < p_t \leq 1 \), which only the working adults \( N_t^1 \) purchase. The children are unable to act independently and the retirees have no incentive to buy credit notes, because they will not be alive in the next period, when the state will pay back its liabilities. The incoming money to the government will be distributed immediately as transfers to the corresponding children, \( p_t T_t = bhn_t^d w_t = \sum_{i=1}^{n_t} bh w_i, \) living in period \( t \). Simultaneously, the government takes in \( t \) revenues from repayments of the former children, who are the workers in \( t \). The state has to pay its liabilities to the retirees in \( t \), arising from the credit notes issued to the workers of the previous period, \( T_{t-1} = \frac{bh n_{t-1}^d w_{t-1}}{p_{t-1}} = \sum_{i=1}^{n_{t-1}} \frac{bh w_i}{p_{t-1}} \). In the next period \( t + 1 \), when the children of \( t \) have become the workers of \( t + 1 \), then these people have to pay back the credited part of their childrearing costs. This will happen via the state as pension payments to their own parents, \( T_t = \frac{bh n_t^d w_t}{p_t} = \sum_{i=1}^{n_t} \frac{bh w_i}{p_t} \), who are the retirees in

\[\text{Since all individuals are homogenous, } n_t \text{ could be interpreted as the net reproduction rate and measures the size of the generation of children relative to the generation of the working adults. The connection between the growth rate of the working generation } m_t \text{ and } n_t \text{ is as follows: } N_t^1 = N_{t+1}^2, \]
\[m_t = \frac{(N_{t+1}^1 - N_{t}^2)}{N_{t}^2} \Rightarrow m_t N_{t}^1 + N_{t}^2 = N_{t+1}^2 \text{ and thus } 1 + m_t = \frac{N_{t+1}^1}{N_{t}^2} = n_t. \]

In the steady state, where \( n_t \) remains constant, the growth rate of the working generation is identically equal to the population growth rate.

\[\text{A diminishing course is relevant, if the succeeding children can use the clothes, the toys and so on of their siblings. Under progressively increasing costs one could imagine time costs of nursing.}\]
Because of the state it is guaranteed that the repayment of the children in accordance with the value of the issued credit notes and therefore consists of the prepaid childrearing costs plus an appropriate rate of return. If $p_t = \frac{1}{1+r_{t+1}}$ or in other words $p_t(1 + r_{t+1}) = 1$, then the credit note issued by the government yields the same return as savings. The repayment from the adult offspring is like a tax, in that the subsequent workers have no choice about it. Hence, the state is able to assure equal returns on credit notes and savings. The ability of the state to enforce the entitlements via corresponding taxation is the important advantage of this system over family-based old-age security systems as described in the model of Cigno/Rosati (1992, pp. 320-321) or Zhang/Zhang (1998, pp.1227-1228). These systems have to rely on the assumption of a family rule or implicit contract to enforce the repayment. Without such an internal rule, on the private level it is impossible to obtain a credit contract where the children are appointed to honor the liabilities. In our model the state plays the role of a sponsor who vouches for the repayment entitlements.

Beyond the linkage of the pension entitlements to the childrearing effort of individuals, a third principle is epitomized by this pension scheme. It also stands for the so-called Hobbes-Rousseau social contract, as described by Samuelson (1958, pp. 479-480). Because the fraction of accountable childrearing costs $b$ is valid for the pensions, the individuals obtain from their own children, as well as for the repayment, the individuals have to provide for their own parents. That is, the young and still working generation will only receive a pension from their own children if she supports the old and currently retired generation (their own parents). The consequence is that it is impossible for the deciding individuals (workers) to cut back the pension payment rate to the retirees of today without cutting back the pension entitlements on their own children, which are represented similarly by the fraction $b$.

In this way, all three simultaneously living generations are connected to each other.

2.2 The households

We want to analyze how the introduction of the above-mentioned pension scheme affects fertility when the fraction of the accountable childrearing costs $b$ is exogenously given by the government. After the predefinition of $b$, this fraction of accountable

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7 If the cost function would be of progressive course, that is if $d > 1$, the repayment to the parents by every single child (which is the average repayment) must rise with the increasing number of children. For this, more children imply a higher per child repayment for the parents.

8 See Zhang/Zhang (1998, p. 1228): Rephrased, the younger generation can only expect to receive a gift, if she gives a similar gift today.
childrearing costs will be the same for all generations.\(^9\)

The individuals have a biological concern about the number of children. The utility function of a representative working individual at period \( t \), born in \( t - 1 \), yields \( U_t(C_t^2, C_{t+1}^3, n_t) \). A possible explicit utility function could have the following form:

\[
U_t = \ln C_t^2 + \frac{1}{1 + \rho} \ln C_{t+1}^3 + \gamma \ln n_t
\]  

(1)

where \( 0 \leq \rho \) is the personal discount rate and \( 0 < \gamma < 1 \) shows the strength of altruism towards the number of children.

The lifetime budget constraint of a working individual, born in \( t - 1 \), is then:

\[
C_t^2 + S_t + \frac{bh w_{t-1}}{p_{t-1}} = (1 - h n_t) w_t
\]  

(2)

\[
C_{t+1}^3 = (1 + r_{t+1}) S_t + \frac{b h n_t w_t}{p_t}
\]  

(3)

Consumption in childhood is determined by one’s parents. Thus, an individual’s consumption in childhood, born in \( t - 1 \), just partially emerges in his own budget constraint as repayment to the individual’s parents. Instead of that, the consumption of the individual’s children fully enters his budget constraint. Equation (3) can be solved for the savings \( S_t \):

\[
S_t = \frac{C_{t+1}^3}{1 + r_{t+1}} - \frac{b h n_t w_t}{p_t (1 + r_{t+1})}
\]  

(4)

By insert equation (4) into equation (2) we obtain the inter-temporal budget constraint:

\[
C_t^2 + \frac{C_{t+1}^3}{1 + r_{t+1}} + \frac{bh w_{t-1}}{p_{t-1}} = \left(1 - h n_t + \frac{b h n_t}{p_t (1 + r_{t+1})}\right) w_t
\]  

(5)

The maximization of (1) subject to (5) yields the Lagrangian function:

\[
L = \ln C_t^2 + \frac{1}{1 + \rho} \ln C_{t+1}^3 + \gamma \ln n_t
\]

\[
+ \lambda \left[ \left(1 - h n_t + \frac{b h n_t}{p_t (1 + r_{t+1})}\right) w_t - C_t^2 - \frac{C_{t+1}^3}{1 + r_{t+1}} - \frac{bh w_{t-1}}{p_{t-1}} \right]
\]

\(^9\)This is comparable to a policy of stable contribution rates, whereby in our model, because of the Hobbes-Rousseau-shaped social contract, the contribution and the entitlement rate are identical and expressed similarly by \( b \).
Through partial differentiation we obtain the first order conditions:

\[
\frac{\partial L}{\partial C_t^2} = \frac{1}{C_t^2} - \lambda = 0 \quad (6)
\]

\[
\frac{\partial L}{\partial C_{t+1}^3} = \frac{1}{1 + \rho} \cdot \frac{1}{C_{t+1}^3} - \frac{\lambda}{1 + r_{t+1}} = 0 \quad (7)
\]

\[
\frac{\partial L}{\partial n_t} = \frac{\gamma}{n_t} - \lambda hw_t + \frac{\lambda}{p_t(1 + r_{t+1})} bw_{t-1} = 0 \quad (8)
\]

\[
\frac{\partial L}{\partial \lambda} = \left(1 - hn_t + \frac{b n_t}{p_t(1 + r_{t+1})}\right) w_t - C_t^2 - \frac{C_{t+1}^3}{1 + r_{t+1}} - \frac{bw_{t-1}}{p_{t-1}} = 0 \quad (9)
\]

The variables \(w_t, r_{t+1}, p_t, p_{t-1}\) and \(w_{t-1}\) are given for the optimizing individuals in period \(t\). From equations (6) and (7) we receive the Euler equation, which determines the relation between consumption in the working period and in retirement:

\[
\frac{C_{t+1}^3}{C_t^2} = \frac{1 + r_{t+1}}{1 + \rho} \quad (10)
\]

Using equations (6), (7) and (8) leads us to the relationships between parental consumption in the working period, old-age consumption and the number of children respectively:

\[
\frac{C_{t+1}^3}{n_t} = \left(1 - \frac{b}{p_t(1 + r_{t+1})}\right) hw_t \quad (11)
\]

\[
\frac{C_{t+1}^2}{n_t} = \frac{(1 + r_{t+1}) \left(1 - \frac{b}{p_t(1 + r_{t+1})}\right) hw_t}{\gamma(1 + \rho)} \quad (12)
\]

As we can see, the higher the rate of altruism towards children \(\gamma\), the larger the number of children relative to consumption \(C_t^2\) and \(C_{t+1}^3\).

By substituting and rearranging and in consideration of \((1 + r_{t+1})p_t = 1\), that is equality of returns of credit notes and savings, we get the optimal number of children per head in period \(t\), depending on the fraction of the accountable childrearing costs \(b\):

\[
n_t = \frac{\gamma(1 + \rho) \left(w_t - \frac{bw_{t-1}}{p_{t-1}}\right)}{[2 + \rho + \gamma(1 + \rho)](1 - b)hw_t} \quad (13)
\]

Since it is not easy to see, we have to examine how the optimal number of children behaves when fraction \(b\) is changed. For this purpose we look at the partial derivative of \(n_t\) according to \(b\):

\[
\frac{\partial n_t}{\partial b} = \frac{\gamma(1 + \rho) \left(w_t - \frac{hw_{t-1}}{p_{t-1}}\right)}{[2 + \rho + \gamma(1 + \rho)]hw_t(1 - b)^2} \quad (14)
\]
As long as the lifetime wage income $w_t$ is larger than the childrearing costs plus interest of that individual in the previous period $\frac{hw_{t-1}}{p_{t-1}}$, the partial derivative $\frac{\partial n_t}{\partial b} > 0$.\(^{10}\)

Taking this condition for granted, this implies that the introduction of the described child-related pension scheme increases the optimal number of children. The reason is that a part of the old-age spending can be financed through the repayment of one’s own children, which increases the marginal benefit of these children. Moreover, the parents have only to bear a part of the childrearing costs of their own children $((1-b)hw_t w_t)$ permanently, if $b > 0$. On the other hand, the parents have to pay back a part of their own childrearing costs. But in comparison to funding alone, the benefits from an additional child obviously can be improved by establishing the proposed new system. Therefore more children are ‘purchased’.

To see further how the pension system works, we have to analyze the effects on savings $S_t$. By solving for $S_t$ and by considering the equation (4) we receive for the optimal savings of an individual, working in period $t$:

$$S_t = \frac{\left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) \left(w_t - \frac{bhw_{t-1}}{p_{t-1}}\right)}{2 + \rho + \gamma(1+\rho)}.$$  \hspace{1cm} (15)

By taking the derivative of $S_t$ according to $b$ we yield:

$$\frac{\partial S_t}{\partial b} = -\frac{\gamma(1+\rho)w_t + [(1-b)^2 - (2-b)b\gamma(1+\rho)] \left(\frac{hw_{t-1}}{p_{t-1}}\right)}{[2 + \rho + \gamma(1+\rho)](1-b)^2}.$$  \hspace{1cm} (16)

As long as $(2-b)b\gamma(1+\rho)$ is smaller than $(1-b)^2$, $\frac{\partial S_t}{\partial b} < 0$ definitely\(^{11}\). That is, the higher the fraction $b$ of the accountable costs of children, the lower the savings accrued by the parents. In this case, the investment in children via credit notes can be seen as a substitute for capital accumulation. Instead of savings, one’s own children are used to pay a part of one’s consumption in retirement. A further effect is that the higher fraction $b$ implies a larger optimal number of children, as stated by equation (14). This in turn reduces the disposable income for consumption and savings, as we see from equation (2). An individual’s repayment to the parents is also larger when $b$ rises. Then fewer resources are available for consumption and savings.

As mentioned above, it is possible to increase the attractiveness of childrearing by

\(^{10}\)If $w_t = \frac{hw_{t-1}}{p_{t-1}}$ the individual would have to spend his whole lifetime income for the repayment of his own childrearing costs. In this case there is nothing left for consumption or rather $C^2_t, C^3_{t+1}$ can be negative, which is not possible.

\(^{11}\)If $(2-b)b\gamma(1+\rho)$ is larger than $(1-b)^2$, then it must hold that $\gamma(1+\rho)w_t > [\left(1-b\right)^2 - (2-b)b\gamma(1+\rho)] \left(\frac{hw_{t-1}}{p_{t-1}}\right)$, such that $\frac{\partial S_t}{\partial b} < 0$.\)
introducing a child-related pension scheme of the suggested form. Other kinds of old-age security such as savings will be substituted by the demand for children.\textsuperscript{12}

\subsection*{2.3 The production sector}

We consider a closed economy with only one production sector and without any technological progress.\textsuperscript{13} Firms produce a unique good, which can be used for consumption $C$ of households or for investments in the capital stock $K$, according to the Cobb-Douglas production function:

$$ Y = AK^\alpha L^{1-\alpha} $$

with labor $L$ and the constant level of technological knowledge $A$. This production function satisfies the well-known classical properties of constant returns to scale ($\lambda F(K, L) = F(\lambda K, \lambda L)$), positive but declining marginal products of capital and labor ($\frac{\partial Y}{\partial K} > 0$, $\frac{\partial^2 Y}{\partial K^2} < 0$, $\frac{\partial Y}{\partial L} > 0$, $\frac{\partial^2 Y}{\partial L^2} < 0$) and the Inada conditions.

The capital stock is owned by the households and depreciates at the constant rate $\delta \geq 0$. The firms have to rent it for production to the rental price $R$ per unit of capital. Hence, the internal interest rate (net rate of return) of a unit of capital is $R - \delta$ for the households. Furthermore, the households can grant loans to each other at the interest rate $r$. Both investments are perfect substitutes with identically equal returns $r = R - \delta$.

The representative firm maximizes its profit according to:

$$ \Pi = AK^\alpha L^{1-\alpha} - wL - (r + \delta)K $$

and there is no time lag between the production and the usage of capital. Therefore, the optimization problem of the representative firm is not inter-temporal. Moreover, firms are in perfect competition and have to take $w$ and $r$ as given. They maximize their profits such that:

$$ \frac{\partial \Pi}{\partial K} = A\alpha k^{\alpha-1} - (r + \delta) = 0 \Rightarrow r = A\alpha k^{\alpha-1} - \delta \quad (17) $$

$$ \frac{\partial \Pi}{\partial L} = A(1-\alpha)k^{\alpha} - w = 0 \Rightarrow w = A(1-\alpha)k^{\alpha}. \quad (18) $$

The wage $w$ and rate of return $R = r + \delta$ equal their marginal products. Because of perfect competition, the sum of factor costs is equal to the total revenues of

\textsuperscript{12}The utility maximum of this household optimization problem does exist for $d = 1$, that is for a linear child cost function. The second order conditions are verified in appendix A.2.

firms and the profits are always zero. This reflects the well-known Euler theorem
\[ Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L. \]
\(^{14}\) Because we assumed a closed economy, no capital transfers with
abroad are considered. Hence, the households’ assets, that is owned by the old at
the start of a period, must be equal to the whole capital stock. The net investment
or in other words the net increase of the capital stock is:\(^ {15}\)

\[ K_{t+1} - K_t = AK_t^\alpha L_t^{1-\alpha} - C_t - \delta K_t \]
\[ K_{t+1} - K_t = w_t L_t + r_t K_t - C_t. \]  

(19)

In the following, we will elaborate on the equilibrium of the economy, which varies
according to the underlying old-age security systems.

### 2.4 The equilibrium of the economy

The optimal behavior of the individuals in the presence of governmental sponsor-
ship was extensively described in subsection (2.1) and (2.2). We trace back to these
optimal solutions and use these now in the framework of the neoclassical growth en-
vironment. In our case of pure consumption costs of childrearing, the labor supply of
individuals remains unaffected. Every individual supplies its labor force inelastically
such that \( N_t^2 = L_t \). Then we can fall back on the economy’s resource constraint
(19). The total consumption in period \( t \) is composed of the consumption of all gen-
erations \( C_t = C_t^2 N_t^2 + C_t^3 N_{t-1}^2 + C_t^1 N_t^2 \). The part \( C_t^2 N_t^2 \) is the consumption of the
working adults at \( t \), who were born in period \( t - 1 \). The part \( C_t^3 N_{t-1}^2 \) denotes the
consumption of the current retirees, who worked in \( t - 1 \) and were born in \( t - 2 \).
The last part is the consumption of children, which equals the childrearing costs
of all parents: \( C_t^1 N_t^2 = h_n w_t N_t^2 \). Because, due to the homogeneity of individuals,
the average fertility equals the individual fertility \( \bar{n}_t = n_t \). We yield for the goods
market equilibrium:

\[ K_{t+1} - K_t = w_t L_t + r_t K_t - C_t^2 N_t^2 - C_t^3 N_{t-1}^2 - h_n w_t N_t^2. \]  

(20)

\(^{14}Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L = (r + \delta)K + wL \Rightarrow Y - \delta K = rK + wL\)

\(^{15}\)See Barro/Sala-i-Martin (1998), pp. 151 f.
Taking into account that $L_t = N_t^2$ and substituting for $C_t^2$ and $C_t^3$ (from the individual budget constraints (2) and (3)) we get:

$$K_{t+1} - K_t = w_t L_t + r_t K_t - \left[ (1 - h n_t) w_t - S_t - \frac{b h w_{t-1}}{p_{t-1}} \right] L_t - h n_t w_t L_t$$

$$- \left[ (1 + r_t) S_{t-1} + \frac{b h n_{t-1} w_{t-1}}{p_{t-1}} \right] L_{t-1}$$

$$K_{t+1} = (1 + r_t) K_t + S_t L_t - (1 + r_t) S_{t-1} L_{t-1} + \frac{b h w_{t-1}}{p_{t-1}} L_t - \frac{b h n_{t-1} w_{t-1}}{p_{t-1}} L_{t-1} .$$

The population develops according to $N_{t+1}^2 = n_t N_t^2$, and since the labor supply is inelastic, the same is true for the labor supply. Then, the goods market condition simplifies to:

$$K_{t+1} = (1 + r_t)(K_t - S_{t-1} L_{t-1}) + S_t L_t . \tag{21}$$

Furthermore, according to Barro/Sala-I-Martin (1998, pp. 151 f.), one can simplify equation (21) by setting the condition $t \geq 2$. Only the older generation owns the whole capital stock, which is exactly the amount of their interest-bearing savings from the previous period. When the economy starts there is an initial capital stock $K_1$, which is owned by the older generation in period 1. Because the old want to end up without any assets when they decease, they sell their entire capital stock to the working generation and consume this amount plus interest during retirement. Furthermore, the old obtain an initial pension in accordance to their assumed childrearing effort. Then, the old consume in period 1: $C_1^3 L_0 = (1 + r_1) K_1 + \frac{b h n_0 w_0}{p_0} L_0$.

Applying this condition to equation (20) yields:16

$$K_2 = (1 + r_1) K_1 + w_1 L_1 - (1 - h n_1) w_1 L_1 + S_1 L_1 + \frac{b h w_0}{p_0} L_1 - h n_1 w_1 L_1$$

$$-(1 + r_1) K_1 - \frac{b h n_0 w_0}{p_0} L_0$$

$$K_2 = S_1 L_1$$

$$K_{t+1} = S_t L_t \quad \forall \quad t \geq 2 \tag{22}$$

We yield for the capital intensity:

$$\frac{K_{t+1}}{L_{t+1}} = \frac{S_t L_t}{L_{t+1}}$$

$$k_{t+1} = \frac{S_t}{n_t} . \tag{23}$$

\[16\text{One would obtain the same result for equation (22) if one assumes } (1 + r_1) S_0 L_0 = (1 + r_1) K_1 \text{ and uses this condition in equation (21).} \]
We can see that the next period capital intensity only depends on the savings of
the currently working adults as well as on their fertility decisions. By substituting
for optimal savings and for the optimal number of children from subsection (2.2) we
obtain the equation for the capital intensity:

\[ k_{t+1} = \frac{\left(1 - b\gamma(1+\rho)\right) \left( w_t - \frac{bhw_{t-1}}{p_{t-1}}\right) \left[2 + \rho + \gamma(1+\rho)\right]}{\gamma(1+\rho) \left(w_t - \frac{bhw_{t-1}}{p_{t-1}}\right)} \cdot \]

This second order linear difference equation simplifies to:

\[ k_{t+1} = \frac{(1 - b - b\gamma(1+\rho))h w_t}{\gamma(1+\rho)} \cdot \]

Since \( w_t = A(1-\alpha)k_t^\alpha \) we yield for the capital intensity:

\[ k_{t+1} = \frac{(1 - b - b\gamma(1+\rho))hA(1-\alpha)k_t^\alpha}{\gamma(1+\rho)} \cdot \] (24)

In the steady state is \( k_{t+1} = k_t = k^* \) and for this we obtain for the steady state
capital intensity: \(^{17}\)

\[ k^* = \left[ \frac{(1 - b - b\gamma(1+\rho))hA(1-\alpha)}{\gamma(1+\rho)} \right]^{\frac{1}{1-\alpha}}. \] (25)

Since we have defined the equilibrium, we can calculate the steady state values
for consumption, savings and fertility. In the steady state is \( w_t = w_{t-1} = w^* \),
\( n_t = n_{t-1} = n^* \) and \( r_{t+1} = r_t = r^* \) \( \Rightarrow 1 + r^* = \frac{1}{\rho^*} \). Then we obtain for the steady
state consumption:

\[ C_t^{2s} = \frac{(1 + \rho)w^* \left(1 - \frac{bh}{\rho^*}\right)}{2 + \rho + \gamma(1+\rho)} \] (26)

\[ C_{t+1}^{2s} = \frac{(1 + r^*)w^* \left(1 - \frac{bh}{\rho^*}\right)}{2 + \rho + \gamma(1+\rho)} \] (27)

and for the steady state fertility:

\[ n^* = \frac{\gamma(1+\rho) \left(1 - \frac{bh}{\rho^*}\right)}{[2 + \rho + \gamma(1+\rho)][(1-b)h]} \cdot \] (28)

\(^{17}\)The dynamics leading to the steady state capital intensity are verified in appendix A.3.
The equilibrium fertility is independent of the labor income $w$ and only varies with the fraction of accountable childrearing costs $b$ and with the interest rate. Finally, we get for the savings:

$$S^* = \frac{\left(1 - \frac{b\gamma(1+\rho)}{1-b}\right)w^* \left(1 - \frac{b\rho}{\gamma}\right)}{2 + \rho + \gamma(1 + \rho)}.$$  \hfill (29)

For the purpose of comparing the steady states with and without the described pension scheme we have to address the comparative statics of the equilibrium.

### 3 Comparative statics of the steady state

First, we consider the steady state capital intensity. As we see from equation (25), $k^*$ is clearly higher if the fraction $b$ of accountable childrearing costs is zero, that is if there is no governmental sponsorship system. For a better understanding let us go to detail and ask how $k^*$ responds to changes in $b$, and look again at the corresponding partial derivative:

$$\frac{\partial k^*}{\partial b} = -\frac{[1 + \gamma(1 + \rho)] \left(1 - b - b\gamma(1 + \rho))h(1 - \alpha)\right)_{\gamma(1 + \rho)}^{1 - \alpha}}{(1 - \alpha)(1 - b - b\gamma(1 + \rho))} < 0.$$  \hfill (30)

Even if $b$ becomes so large that $(1 - b - b\gamma(1 + \rho)) < 0$, it remains: $\frac{\partial k^*}{\partial b} < 0$, because the numerator and the denominator both become negative. Therefore, the steady state capital intensity declines if $b$ increases. Furthermore, there does not exist a certain $b_{k^*_{min}}$ which leads to a smallest reachable capital intensity and which satisfies $\frac{\partial k^*}{\partial b} = 0$. The second derivative $\frac{\partial^2 k^*}{\partial b^2} > 0$ shows that $k^*$ is convex in $b$. The capital intensity is therefore a monotone decreasing function of the fraction of accountable child costs $b$. Thus, the introduction of the sponsorship pension system would decrease the steady state capital intensity $k^*$. The marginal productivity of capital rises, raising the steady state interest rate $r^*$, that is $\frac{\partial r^*}{\partial b} > 0$. The marginal productivity of labor falls, which cuts the equilibrium wage $w^*$, therefore: $\frac{\partial w^*}{\partial b} < 0$.\(^{18}\)

A more detailed way to explain is by recognizing savings and fertility responses to variations in $b$ separately, which justify why capital intensity must sink. How does the equilibrium fertility rate (per capita number of children) react to a changing $b$? The positive response of fertility to an increasing $b$ as shown in section (2.2) proves

---

\(^{18}\)See appendix A.4 for the corresponding derivations.
true even for the large neoclassical economy, since:

\[
\frac{\partial n^*}{\partial b} = \frac{\gamma(1 + \rho)\left[1 - h(1 + r^* + b(1 - b)\frac{\partial r^*}{\partial b})\right]}{[2 + \rho + \gamma(1 + \rho)]h(1 - b)^2}
\]

and as long as \(1 > h(1 + r^* + b(1 - b)\frac{\partial r^*}{\partial b})\), \(\frac{\partial n^*}{\partial b} > 0\). Because \(\frac{1}{\rho^*} = 1 + r^*\) and \(r^* = f(b)\), we must also take into account changes in the steady state interest rate for changes of the equilibrium fertility rate. As shown in appendix A.4, \(\frac{\partial r^*}{\partial b} > 0\), which stems from the capital diluting effect of higher fertility. This is important for the fertility equation, because the higher the equilibrium interest rate, the higher the pension repayment to the retired parents, which reduces the disposable income for childrearing and consumption. But a higher interest rate (which lowers the price of the credit note \(p^*\)), as well as the higher number of children itself, also increases the anticipated pension from an individual’s own children in the next period. That increases the marginal utility of having children. Overall, the introduction of the sponsorship system would raise the steady state fertility rate and increase the future size of the working generation.

The equilibrium savings are negatively affected by a higher fraction of accountable child costs:

\[
\frac{\partial S^*}{\partial b} = -\frac{\left[\frac{\gamma(1 + \rho)}{(1-b)^2} w^* - \frac{\partial w^*}{\partial b} \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right)\right] (1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)}

- \frac{\left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) w^* (h(1 + r^*) + bh\frac{\partial r^*}{\partial b})}{2 + \rho + \gamma(1 + \rho)},
\]

because \(\frac{\partial w^*}{\partial b} < 0 \Rightarrow -\frac{\partial w^*}{\partial b} > 0\) and this is associated with a positive algebraic sign of the corresponding term. Economically, a higher \(b\) increases the optimal number of children and, through larger childrearing costs and larger repayments to the currently retired parents, reduces the disposable income for savings. This in return lowers next period capital intensity and therefore wages. Furthermore, since \(\frac{\partial r^*}{\partial b} > 0\) the last fraction remains negative too. Contrary to intuition, a higher interest rate through a larger \(b\) diminishes savings too. This could be because with a rising interest rate the returns from child-pensions also increases, which makes childrearing more attractive. Therefore, \(\frac{\partial S^*}{\partial b} < 0\), that is savings and children are mutual substitutes as mentioned before in subsection (2.2). Fewer savings lead to a smaller capital stock that will be shared by a larger amount of workers in the following period. Thus, the steady state capital intensity must drop.

\[\text{Since } 0 < h, b < 0 \text{ this is satisfied for adequate values of } h, b.\]
The equilibrium consumption of young and old $C_t^2$ and $C_t^3$ positively depends on the wage income $w^*$, which itself is determined by the steady state capital intensity $k^*$. The introduction of the sponsorship pension system would definitely lower the consumption of the working adults, because of the lowered wage income $\left(\frac{dw^*}{db} < 0\right)$ and the larger repayments to currently retired parents. Conversely, this is not clear for the consumption in retirement since $C_t^3$ also depends on the interest rate and on the pension coming from one’s own children. Because the steady state capital intensity drops, the interest rate will rise, affecting future consumption positively. But the negative income effect of a higher $b$ on savings reduces the amount of savings. This in turn affects old-age consumption negatively. Furthermore, the expected pensions from one’s own children are larger in the sponsorship system. It depends on whether the effects on interest and child pensions compensate for the negative effect on savings.

4 Welfare comparison of child and capital pensions

Now that we have proved the higher steady state fertility rate under governmental sponsorship, the crucial question is whether the individuals can be made better off by the introduction of the new system. We must compare the obtainable maximum utility levels in the steady state with sole funding and with child pensions. If we assume that compulsory fully funding and voluntary private savings yield the same returns, then funding and private savings are perfect substitutes, as described by Breyer (1990, p. 21).\(^{20}\)

The maximum utility in the steady state is defined by:

$$U^* = \ln C_t^2 + \frac{1}{1+\rho} \ln C_{t+1}^3 + \gamma \ln n^* .$$

\(^{20}\)This is easy to deduce by the following simplified considerations: the budget constraint under fully funding in the simplest case is given by: $C_t^2 + S_t = (1 - l_t)w_t$ and $C_{t+1}^3 = x_{t+1} + (1 + r_{t+1})S_t$, $x_{t+1} = (1 + r_{t+1})l_tw_t \Rightarrow S_t = \frac{C_{t+1}^3}{1+r_{t+1}} - l_tw_t$ whereby $l_t$ is the contribution rate to the funded system and $x_{t+1}$ reflects the pension payment. Substituting for $S_t$ in the first equation yield $C_t^2 + \frac{C_{t+1}^3}{1+r_{t+1}} = w_t$, which is exactly the same inter-temporal budget constraint as with only voluntary savings.
By including the according values for consumption and fertility from the equations (26-28) and by considering that \( \frac{1}{\rho} = 1 + r^* \), we obtain for the maximum utility:

\[
U^* = \ln \left( \frac{(1 + \rho)w^*(1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)} \right) + \frac{1}{1 + \rho} \ln \left( \frac{(1 + r^*)w^*(1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)} \right) + \gamma \ln \left( \frac{\gamma(1 + \rho)(1 - bh(1 + r^*))}{[2 + \rho + \gamma(1 + \rho)](1 - bh)} \right).
\]

That accords with the indirect utility function: \( U^*(C^2_t(b), C^3_{t+1}(b), n^*_t(b)) =: V(b) \).

For the comparative static analysis we have to differentiate (each of the arguments of) \( V(b) \) according to \( b \) to observe whether the maximum utility under funding alone is larger than under governmental sponsorship.\(^{21}\)

Then \( \frac{\partial U^*_2(C^2_t)}{\partial b} \) becomes:

\[
\frac{\partial \ln C^2_t}{\partial b} = \frac{\partial w^*}{\partial b} \cdot \frac{1}{w^*} - \left( \frac{h(1 + r^*) + bh \frac{\partial r^*}{\partial b}}{(1 - bh(1 + r^*))} \right) < 0
\]

since \( \frac{\partial w^*}{\partial b} < 0 \) and \( \frac{\partial r^*}{\partial b} > 0 \), as mentioned above. In economic terms, the higher the fraction \( b \) of accountable childrearing costs, the more resources are used to rear children and to settle one’s own liabilities towards one’s parents. This reduces consumption in the working period and therefore the utility from this consumption.

This is not as clear for old-age consumption, where \( \frac{1}{1 + \rho} \frac{\partial U^*_2(C^2_{t+1})}{\partial b} \) becomes:

\[
\frac{1}{1 + \rho} \cdot \left( \frac{\partial r^*}{\partial b} \cdot \frac{1}{1 + r^*} > 0 \right) + \left( \frac{\partial w^*}{\partial b} \cdot \frac{1}{w^*} < 0 \right) + \left( \frac{h(1 + r^*) + bh \frac{\partial r^*}{\partial b}}{(1 - bh(1 + r^*))} > 0 \right) \approx 0.
\]

As already described, the effect on the utility from old-age consumption depends on whether the positive interest effect and the larger child pensions can outweigh the negative effects on wage income and savings.

Finally, we have to consider the influence on the utility from childrearing which is mapped by \( \gamma \frac{\partial U^*_3(n^*_t)}{\partial b} \):

\[
\gamma \frac{\partial \ln n^*_t}{\partial b} = \gamma \left[ \frac{1 - h \left( 1 + r^* + b(1 - b) \frac{\partial r^*}{\partial b} \right)}{(1 - b)(1 - bh(1 + r^*))} \right] > 0
\]

if \( 1 > h \left( 1 + r^* + b(1 - b) \frac{\partial r^*}{\partial b} \right) \). Then, the introduction of the sponsorship system is associated with more children and thus with a larger utility from childrearing.

\(^{21}\)See appendix A.4 for the corresponding derivations.
At this point, no overall statement is possible regarding the welfare improvement through the introduction of the proposed system, since on this theoretical level one cannot state which effects would be predominant. But if the loss of utility from the lowered consumption in the working period exceeds the additional utility from the larger old-age consumption and from the larger number of children, then saving remains associated with the higher maximum utility for the individuals.

5 Simulations

To underpin the above statements we wish to provide some numerical results. For that we need some assumptions about the values of the necessary parameters. The length of a period $t$ should be 25 years. First, we make some assumptions about the production sector. The period depreciation rate is $\delta = 0.72$, which corresponds to a yearly rate of 5%. The profit share of $\alpha = 0.33$ is also a common assumption. The level of technological knowledge is $A = 30$. Second, we have to determine the characteristics of the utility function. The time preference parameter is $\beta = 0.064$ or 2% per year. The strength of altruism shall be $\gamma = 0.33$, which emphasizes the downgrading of the importance of children for the parents. Third, we have to determine the course of the child cost function, which shall be linear and with $h = 0.2$. This means that a single child would make demands on 20% of the wage income of parents.

To accomplish a comparison between situations with and without child-related pensions for all relevant variables, we determine the fraction of accountable childrearing costs by $b = 0$ and $b = 0.3$ respectively. We receive the following results, which fully corroborate the above-mentioned theoretical results:

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & capital pension, $b = 0$ & & & & & \\
\hline
$k$ & $w$ & $r$ & $S$ & $C^2$ & $C^3$ & $n$ & $U$ \\
\hline
19.94 & 53.97 & 0.61 & 16.96 & 27.82 & 27.36 & 0.851 & 5.29 \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & child pension, $b = 0.3$ & & & & & \\
\hline
$k$ & $w$ & $r$ & $S$ & $C^2$ & $C^3$ & $n$ & $U$ \\
\hline
7.90 & 39.75 & 1.76 & 8.01 & 17.10 & 28.77 & 1.014 & 4.89 \\
\hline
\end{tabular}

\footnote{For common values of parameters such as $\alpha$, $\delta$, $\rho$ or others see for example Barro/Sala-I-Martin (1998, pp. 152, 155).}
As we can easily see, the introduction of the proposed child-related pension system increases the optimal number of children despite the relatively strong downgrading of the utility from children. In this case, when only savings provide old-age security, the desired number of children is not sufficient to fully replace the parental generation. On the other hand, with a child pension, when only 30% of the childrearing costs can be credited for old-age security, we achieve population growth. Moreover, through the child-related pensions old-age consumption increases too. But introducing the child-related pension scheme reduces the steady state savings rate \( s = S/y \) of the economy from \( s_F = 0.21 \) under funding to \( s_C = 0.13 \) with child-related pensions. The higher population growth accompanied with a lower savings rate can be visualized as follows:

**Figure 1**

![Diagram showing dynamics to the equilibrium](image)

By introducing the child-related pension system we achieve a new steady state with a lower capital intensity \( k_C \). Despite the obtainable higher population growth, there is a negative side. In this neoclassical growth environment, the introduction of the child-related pension system would be associated with a smaller overall utility for the individuals compared with funding alone. This is, the additional utility from children and old-age consumption cannot fully compensate the loss of utility from the lower consumption during the working period. Since utility matters for the wellbeing of individuals, in this specific case the introduction of the proposed new system would have to be refused. But, clearly, choosing another economic environment such as modern idea-based growth models like that of Jones (1995, 2001) will probably lead us to the contrary result of increasing utility, because in these models wage income depends positively on the population growth rate.
Moreover, we can analyze the steady state variables as functions of the fraction of accountable childrearing costs $b$. Figure 2 visualizes this for the key variables $k^*(b), U^*(b), n^*(b)$:

As already mentioned in the analytical part, $k^*(b)$ continual decreases as $b$ rises, since the higher steady state fertility rate as well as the lower savings lead to the capital dilution effect, which in turn reduces the wage income of households for consumption, savings and fertility in the next period. In contrast, the steady state fertility rate $n^*(b)$ first increases with $b$ up to a maximum level. But at some point in time, the negative effects on the disposable income for consumption and fertility start to predominate, due to capital dilution and larger repayments to parents. Then $n^*(b)$ decreases with a further increasing fraction $b$. Because parents have to take $b$ as given (state-defined), they try to maximize their utility by choosing the fertility rate in each specific situation with the given fraction of accountable childrearing costs $b$. But they always reach a lower maximum utility level. Therefore, the maximum utility level $U^*(b)$ is a decreasing function of $b$ because the loss of utility from the lower consumption during the working period rises above the additional utility from old-age consumption and fertility. As a consequence, if individuals could decide about the child-related pension system, in the neoclassical environment they would reject it in favour of fully funding.
Furthermore, it is of interest which role the degree of altruism plays in the contribution of the child-related pension system to population growth. In figure 3 we can observe how the degree of altruism changes the course of the steady state fertility rate as a function of $b$:

![Figure 3](image)

The smaller the degree of parental altruism, the larger the positive effect of the proposed pension scheme on the fertility rate, measured in the maximum difference $\Delta_{\text{max}}n^*_{\gamma_i}(\gamma_i, b) - n^*(\gamma_i, 0)$. For example, for $\gamma = 0.2$ the fertility rate increases from $n^*(0.2, 0) = 0.5526$ to $n^*_{\text{max}}(0.2, 0.5) = 0.8085$ and the difference is $\Delta_{\text{max}}n^*_{0.2} = n^*_{\text{max}}(0.2, 0.5) - n^*(0.2, 0) = 0.2559$. Contrary to that, if the degree of altruism is larger, for example $\gamma = 0.5$, then the difference would only be $\Delta_{\text{max}}n^*_{0.5} = 0.10$. That is, with an increasing degree of altruism, the fraction $b$, which maximizes the steady state fertility rate, will decrease.

Especially in the case where parental altruism towards children is only weakly developed, the child-related pension system provides the best way of increasing the equilibrium fertility rate. But as already elucidated, the overall individual utility level in the steady state always drops with an increasing fraction of accountable childrearing costs.

### 6 Conclusion

As widely discussed in the literature, the underlying pension scheme (PAYG or fully funding) generates or at least can generate a negative effect on the incentive to have children. As a result, the introduction of such systems would reduce the optimal
number of children. It was our aim to look for an alternative pension system that, in coexistence with private capital savings, would not generate this deficiency. We showed that it is possible to create a pension scheme which is associated with a larger optimal number of children for the deciding households in comparison to funding alone. As already stated, this results from the direct connection of pension entitlements of parents to the childrearing costs incurred by those parents. In this way, and contrary to the usual PAYG system, the individuals not only bear the childrearing costs, but also yield the returns directly related to the childrearing effort expended on their own children. Thus, there is no longer a positive externality to the society as in the PAYG system, which suppresses the incentive to have own children. Furthermore, we verified this result even in the neoclassical growth environment and obtained the expected results. The introduction of the child-related pension system exhibits a positive fertility effect, despite declining wage income caused by a decreasing capital intensity. But, as mentioned above, compared to fully funding, the higher population growth is connected with a smaller overall utility for the individuals in this neoclassical growth environment, where all growth is due to capital accumulation. Further research would be required to incorporate the described child-related pension scheme into an endogenous growth model, such as those of Romer (1990), and Jones (1995, 2001). Then one could expect to obtain a positive effect of that pension system on the individual’s maximum utility in the steady state, because there the development of the wage income is positively related with the population growth rate.

References


## A Appendix

### A.1 Scheme of transfers

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<tr>
<th>Child in $t-1$</th>
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<th>Child in $t+1$</th>
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<td>$C_{t+1}^{1i}$</td>
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<td>$\sum_{i=1}^{n_{t+1}} \frac{bh\bar{w}<em>{t+1}}{p</em>{t+1}}$</td>
</tr>
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A.2 Second order conditions

We have to check whether or not there is a maximum of the utility function. For this purpose we analyze the corresponding Hessian matrix. Because we assumed \( p_t(1 + r_{t+1}) = 1 \) throughout the paper, we will use this to simplify the following analysis of the second order conditions. The Hessian matrix is given by:

\[
H = \begin{pmatrix}
L_{\lambda\lambda} = 0 & L_{\lambda C_t^2} = -\frac{1}{(1+r_{t+1})^2} & L_{\lambda C_3^2} = -\frac{1}{(1+r_{t+1})^2} & L_{\lambda n_t} = -\frac{1}{(1-r_{t+1})^2} \\
L_{C_t^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_t^2 C_t^2} = 0 & L_{C_t^2 C_3^2} = 0 & L_{C_t^2 n_t} = 0 \\
L_{C_3^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_3^2 C_t^2} = 0 & L_{C_3^2 C_3^2} = 0 & L_{C_3^2 n_t} = 0 \\
L_{n_t\lambda} = -(1-b)hw_t & L_{n_t C_t^2} = 0 & L_{n_t C_3^2} = 0 & L_{n_t n_t} = -(n_t)^2
\end{pmatrix}
\]

Because the Hessian matrix evaluates a maximum/minimum at the stationary point, we can use the first order conditions to simplify the entries of the matrix. First, \((1-b)hw_t, w_t = \gamma C_t^2 \Rightarrow -(1-b)hw_t = -\frac{\gamma C_t^2}{n_t}\). Second, because of the Euler equation (10) we yield \(-\frac{1}{(1+\rho)(C_t^2)^2} = -\frac{1+\rho}{((1+r_{t+1})C_t^2)^2}\). Then, the Hessian matrix becomes:

\[
H = \begin{pmatrix}
L_{\lambda\lambda} = 0 & L_{\lambda C_t^2} = -\frac{1}{(1+r_{t+1})^2} & L_{\lambda C_3^2} = -\frac{1}{(1+r_{t+1})^2} & L_{\lambda n_t} = -\frac{\gamma C_t^2}{n_t} \\
L_{C_t^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_t^2 C_t^2} = 0 & L_{C_t^2 C_3^2} = 0 & L_{C_t^2 n_t} = 0 \\
L_{C_3^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_3^2 C_t^2} = 0 & L_{C_3^2 C_3^2} = 0 & L_{C_3^2 n_t} = 0 \\
L_{n_t\lambda} = \gamma C_t^2 & L_{n_t C_t^2} = 0 & L_{n_t C_3^2} = 0 & L_{n_t n_t} = -(n_t)^2
\end{pmatrix}
\]

For a maximum, that we are looking for, it is necessary that the determinant of the bordered Hessian matrix of the Lagrangian function has the algebraic sign: \((-1)^k\), \(k\) arguments of the objective function. In our case the sign has to be \((-1)^3 = -1 < 0\).

The largest \(k - 1\) leading principal minors have to alternate in sign, that is, the 3rd leading principal minor, which is the determinant of the 3rd leading principal submatrix, has to be \(> 0\) and the 4th leading principal minor, that is the determinant of the Hessian matrix, has to be \(< 0\).\(^{23}\) We get for the 3rd leading principal minor:

\[
\begin{vmatrix}
L_{\lambda\lambda} = 0 & L_{\lambda C_t^2} = -\frac{1}{(1+r_{t+1})^2} & L_{\lambda C_3^2} = -\frac{1}{(1+r_{t+1})^2} \\
L_{C_t^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_t^2 C_t^2} = 0 & L_{C_t^2 C_3^2} = 0 \\
L_{C_3^2\lambda} = -\frac{1}{(1+r_{t+1})^2} & L_{C_3^2 C_t^2} = 0 & L_{C_3^2 C_3^2} = 0
\end{vmatrix}
= \frac{1}{((1+r_{t+1})C_t^2)^2} + \frac{1+\rho}{((1+r_{t+1})C_t^2)^2} = \frac{2+\rho}{((1+r_{t+1})C_t^2)^2} > 0.
\]

\(^{23}\)See Klein (1998), p. 330
To calculate the determinant of the Hessian matrix (|H|) we can develop this determinant according to the second row (Laplace expansion) and ascertain:

\[
|H| = (-1) \cdot (-1)^{(2+1)} \cdot \begin{vmatrix}
L_{\lambda}\frac{c^2}{t} & -1 & L_{\lambda}\frac{c^3}{t+1} & \frac{\gamma c^2}{n_t} \\
L_{\lambda}\frac{c^3}{t+1} & 0 & L_{\lambda}\frac{c^3}{t+1} & \frac{1+\rho}{((1+r_{t+1})c^2)^2} \\
L_{\lambda}n_t & 0 & L_{\lambda}n_t & 0 \\
L_{\lambda}n_t & 0 & L_{\lambda}n_t & 0
\end{vmatrix}
\]

\[
+ \left( \frac{1}{(C^2_t)^2} \right) \cdot (-1)^{(2+2)} \cdot \begin{vmatrix}
L_{\lambda}\frac{\alpha^2}{t} & L_{\lambda}\frac{\alpha^3}{t+1} & \frac{\gamma (1+\rho)}{(1+r_{t+1})} & \frac{\gamma \gamma^2 (1+\rho)}{(1+r_{t+1})C^2_t n_t} \\
L_{\lambda}\frac{\alpha^3}{t+1} & 0 & L_{\lambda}\frac{\alpha^3}{t+1} & \frac{\gamma (1+\rho)}{(1+r_{t+1})c^2} \\
L_{\lambda}n_t & 0 & L_{\lambda}n_t & 0 \\
L_{\lambda}n_t & 0 & L_{\lambda}n_t & 0
\end{vmatrix}
\]

\[
= -\frac{\gamma (1+\rho)}{(1+r_{t+1})C^2_t n_t} - \frac{\gamma + \gamma^2 (1+\rho)}{(1+r_{t+1})C^2_t n_t)^2} = -\frac{\gamma (1+\rho)(1+\gamma) + \gamma}{(1+r_{t+1})C^2_t n_t^2} < 0 .
\]

Therefore, we can state that there truly exists a local maximum evaluated at the stationary point. Furthermore, because \( d = 1 \), the constraint (5) is linear. Since the utility function (1) is strictly concave, the local maximum equals the global maximum.

### A.3 Stability of the steady state

To show analytically the stability of the equilibrium we have to solve the fundamental difference equation (24). For this purpose we simplify \( \Omega = \frac{(1-b-b(1+\rho))b A(1-\alpha)}{\gamma (1+\rho)} \) and linearize the equation by taking the logs and yield:

\[
\ln k_{t+1} = \alpha \ln k_t + \ln \Omega .
\]

Further on, we set \( z_{t+1} = \ln k_{t+1} \), \( \tau = \ln \Omega \), \( z_t = \ln k_t \). We obtain a first order linear difference equation that can be solved by iteration. Defining the initial period \( t = 0 \) with a given initial value \( z_0 > 0 \) the equation becomes:

\[
\begin{align*}
z_1 &= \alpha z_0 + \tau \\
z_2 &= \alpha z_1 + \tau = \alpha (\alpha z_0 + \tau) + \tau = \alpha^2 z_0 + \alpha \tau + \tau \\
z_3 &= \alpha z_2 + \tau = \alpha (\alpha^2 z_0 + \alpha \tau + \tau) = \alpha^3 z_0 + \alpha^2 \tau + \alpha \tau + \tau \\
\vdots \\
z_t &= \alpha^t z_0 + \tau \cdot \sum_{i=0}^{t-1} \alpha^i .
\end{align*}
\]
The second part on the right hand side, $S_G = \tau \sum_{i=0}^{t-1} \alpha^i$, can be calculated by subtracting $\alpha S_G$ from the sum $S_G$:

$$
S_G = \tau + \alpha \tau + \alpha^2 \tau + \alpha^3 \tau + \ldots + \alpha^{t-1} \tau
$$

$$
-\alpha S_G = \alpha \tau + \alpha^2 \tau + \alpha^3 \tau + \ldots + \alpha^{t-1} \tau + \alpha^t \tau
$$

$$
(1 - \alpha)S_G = \tau(1 - \alpha^t)
$$

$$
S_G = \tau \cdot \left( \frac{1 - \alpha^t}{1 - \alpha} \right).
$$

Then we obtain the simplified solution of this difference equation.

$$
z_t = \alpha^t z_0 + \tau \left( \frac{1 - \alpha^t}{1 - \alpha} \right)
$$

Backward transformation of the variables and removal of logarithms yields:

$$
\ln k_t = \alpha^t \ln k_0 + \ln \Omega \left( \frac{1 - \alpha^t}{1 - \alpha} \right)
$$

$$
k_t = k_0^{\alpha^t} \Omega \left( \frac{1 - \alpha^t}{1 - \alpha} \right).
$$

(38)

Since $0 < \alpha < 1$, for $t \to \infty$ it follows that $\alpha^t \to 0$ and thus $k_0^{\alpha^t} \to 1$. In this way, with increasing $t$ equation (38) approaches the term $\Omega \left( \frac{1 - \alpha^t}{1 - \alpha} \right)$, which itself converges to $\Omega \left( \frac{1}{1 - \alpha} \right)$. The backward transformation of $\Omega$ leads to:

$$
k_t = k_0^{\alpha^t} \left[ \frac{(1 - b - b \gamma(1 + \rho)) h A (1 - \alpha)}{\gamma(1 + \rho)} \right]^{\frac{1 - \alpha^t}{1 - \alpha}}.
$$

Because $k_0^{\alpha^t} \to 1$ and $\left[ \frac{(1 - b - b \gamma(1 + \rho)) h A (1 - \alpha)}{\gamma(1 + \rho)} \right]^{\frac{1 - \alpha^t}{1 - \alpha}} \to \left[ \frac{(1 - b - b \gamma(1 + \rho)) h A (1 - \alpha)}{\gamma(1 + \rho)} \right]^{\frac{1}{1 - \alpha}}$, $k_t \to k^*$.

In other words:

$$
\lim_{t \to \infty} k_t = \lim_{t \to \infty} \left( k_0^{\alpha^t} \left[ \frac{(1 - b - b \gamma(1 + \rho)) h A (1 - \alpha)}{\gamma(1 + \rho)} \right]^{\frac{1 - \alpha^t}{1 - \alpha}} \right) = k^*.
$$

Regardless of the starting point $k_0 > 0$, the capital intensity $k_t$ always converges with increasing $t$ to the steady state capital intensity $k^*$ as in equation (25). Therefore, the steady state does exist and is stable.
A.4 Derivations of steady state variables according to $b$

1. The steady state interest rate is determined by the steady state capital intensity:

$$r^* = A\alpha k^{*\alpha - 1} - \delta$$

$$= A\alpha \left[ \frac{(1 - b - b\gamma(1 + \rho))hA(1 - \alpha)}{\gamma(1 + \rho)} \right]^\frac{1}{1-\alpha} \alpha^{(\alpha-1)} - \delta.$$ 

Since, for $\alpha \neq 1$, $\frac{\alpha - 1}{1 - \alpha} = -1$ we yield:

$$r^* = A\alpha \frac{1}{(1 - b - b\gamma(1 + \rho))hA(1 - \alpha)} - \delta$$

$$= \frac{A\alpha \gamma(1 + \rho)}{(1 - b - b\gamma(1 + \rho))hA(1 - \alpha)} - \delta$$

$$r^* = \frac{\alpha \gamma(1 + \rho)}{(1 - \alpha)(1 - b - b\gamma(1 + \rho))h} - \delta.$$ 

Response of the steady state interest rate to variations of $b$:

$$\frac{\partial r^*}{\partial b} = \frac{0 \cdot (1 - \alpha)(1 - b - b\gamma(1 + \rho))h - \alpha \gamma(1 + \rho)[-h(1 - \alpha) - \gamma(1 + \rho)h(1 - \alpha)]}{[(1 - \alpha)(1 - b - b\gamma(1 + \rho))h]^2}$$

$$= \frac{\alpha \gamma(1 + \rho)h(1 - \alpha) + \alpha \gamma(1 + \rho)\gamma(1 + \rho)h(1 - \alpha)}{[(1 - \alpha)(1 - b - b\gamma(1 + \rho))h]^2}$$

$$= \frac{\alpha \gamma(1 + \rho) + \alpha \gamma(1 + \rho)\gamma(1 + \rho)}{(1 - \alpha)h[(1 - b - b\gamma(1 + \rho))]^2}$$

$$\frac{\partial r^*}{\partial b} = \frac{\alpha \gamma(1 + \rho)(1 + \gamma(1 + \rho))}{(1 - \alpha)h[(1 - b - b\gamma(1 + \rho))]^2} > 0.$$ 

Because $\frac{1}{\rho} = 1 + r^*$ it follows that $\frac{\partial p^*}{\partial b} < 0$.

2. The steady state wage income $w^*$ is defined as well through $k^*$:

$$w^* = A(1 - \alpha)k^{*\alpha}$$

$$= A(1 - \alpha) \left[ \frac{(1 - b - b\gamma(1 + \rho))hA(1 - \alpha)}{\gamma(1 + \rho)} \right]^\frac{1}{1-\alpha} \alpha^{-\alpha}$$

$$= A(1 - \alpha)[A(1 - \alpha)]^{\frac{1}{1-\alpha}} \left[ \frac{(1 - b - b\gamma(1 + \rho))h}{\gamma(1 + \rho)} \right]^\frac{-\alpha}{1-\alpha}.$$ 

Since $1 + \frac{\alpha}{1 - \alpha} = \frac{1}{1 - \alpha}$ we then get:

$$w^* = [A(1 - \alpha)]^{\frac{1}{1-\alpha}} \left[ \frac{(1 - b - b\gamma(1 + \rho))h}{\gamma(1 + \rho)} \right]^\frac{-\alpha}{1-\alpha}.$$
The equilibrium wage income is affected by $b$ as follows:

\[
\frac{\partial w^*}{\partial b} = \left( \frac{\alpha}{1-\alpha} \right) \left[ \frac{(1 - b - b\gamma(1 + \rho))}{\gamma(1 + \rho)} \right] \left( \frac{\alpha}{1-\alpha} \right) \left[ \frac{(1 + \gamma(1 + \rho))}{[\gamma(1 + \rho)]^2} \right] \frac{[A(1 - \alpha)]^{\frac{1}{1-\alpha}}}{[1 - b - b\gamma(1 + \rho)]}
\]

\[
= -\frac{\alpha(1 + \gamma(1 + \rho))}{(1 - \alpha)(1 - b - b\gamma(1 + \rho))} \left[ \frac{(1 - b - b\gamma(1 + \rho))}{\gamma(1 + \rho)} \right] \left( \frac{\alpha}{1-\alpha} \right) \left[ \frac{(1 + \gamma(1 + \rho))}{[\gamma(1 + \rho)]^2} \right] \frac{[A(1 - \alpha)]^{\frac{1}{1-\alpha}}}{[1 - b - b\gamma(1 + \rho)]}
\]

Even if $1 - b - b\gamma(1 + \rho) < 0$, the numerator as well as the denominator become negative, so that the algebraic sign of the whole fraction does not alter.

3. The steady state fertility rate $n^*$ that is defined by equation (28) reacts to changes in $b$. Since $\frac{1}{\rho} = 1 + r^*$ we can redefine $n^*$ as:

\[
n^* = \frac{\gamma(1 + \rho)(1 - bh(1 + r^*))}{[2 + \rho + \gamma(1 + \rho)](1 - b)h}
\]

and because $r^* = f(b)$ we have to consider changes in $r^*$ too, while differentiating $n^*$. We then get:

\[
\frac{\partial n^*}{\partial b} = \frac{\gamma(1 + \rho)(-h(1 + r^*) - bh\frac{\partial r^*}{\partial b})}{[2 + \rho + \gamma(1 + \rho)]^2(1 - b)^2h^2}
\]

\[
-\left[ \frac{2 + \rho + \gamma(1 + \rho)h\gamma(1 + \rho)(1 - bh(1 + r^*))}{[2 + \rho + \gamma(1 + \rho)]^2(1 - b)^2h^2} \right]
\]

\[
= \frac{\gamma(1 + \rho)(-h(1 + r^*) - bh\frac{\partial r^*}{\partial b})(1 - b) + \gamma(1 + \rho)(1 - bh(1 + r^*))}{[2 + \rho + \gamma(1 + \rho)]h(1 - b)^2}
\]

\[
= \frac{\gamma(1 + \rho) - \gamma(1 + \rho)h(1 + r^*) - \gamma(1 + \rho)hb(1 - b)\frac{\partial r^*}{\partial b}}{[2 + \rho + \gamma(1 + \rho)]h(1 - b)^2}
\]

\[
\frac{\partial n^*}{\partial b} = \frac{\gamma(1 + \rho)\left[1 - h\left(1 + r^* + b(1 - b)\frac{\partial r^*}{\partial b}\right)\right]}{[2 + \rho + \gamma(1 + \rho)]h(1 - b)^2}
\]
4. The equilibrium per capita savings $S^*$ that are defined by equation (29) are affected by $b$ as follows: Since $\frac{1}{\rho^*} = 1 + r^*$ we can redefine $S^*$ as:

$$S^* = \frac{\left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) w^* (1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)} \Theta = \frac{\Theta}{\Upsilon}.$$  

Furthermore, we have to consider again $r^* = f(b)$, but also $w^* = g(b)$, and can deduce the partial derivative with the following steps:

First, by taking into account the product rule we partially differentiate the numerator of that fraction and get:

$$\frac{\partial \Theta}{\partial b} = -\frac{\gamma(1 + \rho)}{(1-b)^2} w^* (1 - bh(1 + r^*)) + \frac{\partial w^*}{\partial b} \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) (1 - bh(1 + r^*))$$

$$- \left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right) w^* \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right)$$

$$= \left[ \frac{\partial w^*}{\partial b} \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) - \frac{\gamma(1+\rho)}{(1-b)^2} w^* \right] (1 - bh(1 + r^*))$$

$$- \left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right) w^* \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right).$$

Second, in compliance with the quotient rule we have to differentiate the whole fraction by:

$$\frac{\partial S^*}{\partial b} = \frac{\frac{\partial \Theta}{\partial b} \cdot \Upsilon - \Theta \cdot \frac{\partial \Upsilon}{\partial b}}{\Upsilon^2} = \frac{\frac{\partial \Theta}{\partial b} \cdot \Upsilon - \Theta \cdot 0}{\Upsilon^2} = \frac{\frac{\partial \Theta}{\partial b}}{\Upsilon}$$

and we obtain:

$$\frac{\partial S^*}{\partial b} = \frac{\left[ \frac{\partial w^*}{\partial b} \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) - \frac{\gamma(1+\rho)}{(1-b)^2} w^* \right] (1 - bh(1 + r^*)) - \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) w^* \left(h(1 + r^*) + bh \frac{\partial r^*}{\partial b}\right)}{2 + \rho + \gamma(1 + \rho)}$$

$$= \frac{- \left(1 - \frac{b\gamma(1+\rho)}{1-b}\right) w^* \left(h(1 + r^*) + bh \frac{\partial r^*}{\partial b}\right)}{2 + \rho + \gamma(1 + \rho)}.$$
A.5 Derivation of the steady state utility according to \( b \)

For the steady state utility we revert to equation (34):

\[
U^* = \ln \left( \frac{(1 + \rho)w^*(1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)} \right) + \frac{1}{1 + \rho} \ln \left( \frac{(1 + r)w^*(1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)} \right) \\
+ \gamma \ln \left( \frac{\gamma(1 + \rho)(1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)(1 - b)h} \right).
\]

Since \( \frac{d \ln f(x)}{dx} = \frac{df(x)}{dx} f(x) \) we get for the several parts of this function:

\[
\frac{\partial \ln C_i^{2*}}{\partial b} = \frac{\partial C_i^{2*}}{C_i^{2*}} \\
\frac{\partial C_i^{2*}}{\partial b} = (1 + \rho) \left[ \frac{\partial w^*}{\partial b} (1 - bh(1 + r^*)) + w^* (-h(1 + r^*) - bh \frac{\partial r^*}{\partial b}) \right] \frac{(2 + \rho + \gamma(1 + \rho))}{(2 + \rho + \gamma(1 + \rho))^2} \\
\frac{\partial C_i^{2*}}{C_i^{2*}} \frac{\partial b}{\partial C_i^{2*}} = (1 + \rho) \left[ \frac{\partial w^*}{\partial b} (1 - bh(1 + r^*)) - w^* (h(1 + r^*) + bh \frac{\partial r^*}{\partial b}) \right] \frac{2 + \rho + \gamma(1 + \rho)}{(1 + \rho)w^*(1 - bh(1 + r^*))} \\
= \frac{\partial w^*}{\partial b} (1 - bh(1 + r^*)) - w^* \left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right) \frac{w^*(1 - bh(1 + r^*))}{w^*(1 - bh(1 + r^*))} \\
\Rightarrow \frac{\partial \ln C_i^{2*}}{\partial b} = \frac{\partial w^*}{\partial b} \frac{1}{w^*} - \frac{(h(1 + r^*) + bh \frac{\partial r^*}{\partial b})}{(1 - bh(1 + r^*))} < 0
\]

since \( \frac{\partial w^*}{\partial b} < 0 \) and \( \frac{\partial r^*}{\partial b} > 0 \).
\[
\frac{1}{1 + \rho} \frac{\partial \ln C_{t+1}^3}{\partial b} = 1 \frac{\partial C_{t+1}^3}{\partial b}
\]

\[
\frac{\partial C_{t+1}^3}{\partial b} = \frac{(1 + r^*) \left[ \frac{\partial w^*}{\partial b} (1 - bh(1 + r^*)) - w^* \left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right) \right] + \frac{\partial r^*}{\partial b} w^* (1 - bh(1 + r^*))}{2 + \rho + \gamma(1 + \rho)}
\]

\[
\frac{1}{1 + \rho} \frac{\partial C_{t+1}^3}{\partial b} = \frac{(1 + r^*) \left[ \frac{\partial w^*}{\partial b} (1 - bh(1 + r^*)) - w^* \left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right) \right] + \frac{\partial r^*}{\partial b} w^* (1 - bh(1 + r^*))}{(1 + \rho)(2 + \rho + \gamma(1 + \rho))}
\]

\[
\Rightarrow \quad \frac{1}{1 + \rho} \frac{\partial \ln C_{t+1}^3}{\partial b} = \frac{1}{1 + \rho} \left( \frac{\partial r^*}{\partial b} \frac{1}{1 + r^*} > 0 + \frac{\partial w^*}{\partial b} \frac{1}{w^*} < 0 - \frac{\left( h(1 + r^*) + bh \frac{\partial r^*}{\partial b} \right)}{(1 - bh(1 + r^*))} \right) \iff 0
\]

\[
\gamma \frac{\partial \ln n_t^*}{\partial b} = \frac{\partial n_t^*}{\partial b} n_t^* = \gamma(1 + \rho) \left[ 1 - h \left( 1 + r^* + b(1 - b) \frac{\partial r^*}{\partial b} \right) \right] \cdot \frac{[2 + \rho + \gamma(1 + \rho)](1 - b)h}{\gamma(1 + \rho)(1 - bh(1 + r^*))}
\]

\[
\Rightarrow \quad \gamma \frac{\partial \ln n_t^*}{\partial b} = \gamma \frac{\left[ 1 - h \left( 1 + r^* + b(1 - b) \frac{\partial r^*}{\partial b} \right) \right]}{(1 - b)(1 - bh(1 + r^*))} > 0
\]

if \( 1 > h \left( 1 + r^* + b(1 - b) \frac{\partial r^*}{\partial b} \right) \).
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