The Economic Consequences of Immigration and why a Neoclassical Model will Fail

by Manfred Jäger
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When immigration is analyzed using standard neoclassical models with all markets working perfectly then it is beneficial for natives as a group. This paper shows that this result rests on the edge of the knife: The positive effects will generically be dominated in models that allow for imperfections even if these imperfections are small. In this sense the neoclassical analysis is structurally unstable when applied to immigration.

Key words: Immigration, Immigration Surplus, General Equilibrium Model, Structural Instability
JEL classification: F22, D50

1 Introduction

In aggregation, natives gain from immigration. This claim holds true in standard neoclassical models. The paper of Berry and Soligo (1969) proved this argument within a very simple setting. Individuals receive their marginal product as labor income which is smaller than the average product. So, immigration benefits natives, since immigrants receive less than they contribute. The result can be generalized to a large extent. The bottom line is that as long as immigrants differ from natives the latter gain from immigration. Indeed, immigration enlarges the possible gains from exchange and thereby contribute to aggregate economic welfare.

This paper studies quantitative properties of the economic consequences of immigration and shows that the Berry – Soligo result rests on the edge of the knife: If the economy deviates only slightly from the perfectly competitive one then the force present in the analysis of Berry and Soligo becomes generically dominated by effects caused by
imperfections. Figure 1 illustrates this argument. Consider the diagram on the left. The parabola shows the income effect of immigration as analyzed by Berry and Soligo. On the abscise we draw the immigration ratio \( m = L_I/L \) and on the ordinate the percentage change of income of natives (immigration surplus). The most important property of this curve is its zero-slope at origin. We will see that if we allow for market imperfections additional effects of immigration appear that are (in general) *not flat at the origin*, i.e. the corresponding diagram looks like the one on the right in figure 1. Consequently immigration will have negative effects to income of natives if immigration is small.

![Figure 1: First-Order and Second-Order Effects.](image)

Mathematically speaking, the effect found in the Berry – Soligo case is of second-order. If we leave perfect markets, additional effects appear that are in general of first-order and therefore dominate the second-order effects. We draw the conclusion that the Berry – Soligo approach is *structurally* unstable. Already with minor imperfections the predicted consequences of immigration change. Section 2 will discuss the approach in a rather abstract manner. This analysis allows us to identify the general mechanism behind the structural instability. Section 3 demonstrates via back-of-the-envelope calculation that the argument is not a theoretical curiosity but a severe, quantitative problem. The analysis of this paper concentrates on income effects. Because of of Ostroy’s Theorem (1980) this analysis extends to welfare as is sketched in section 3.3. The 4th section concludes.

*In section 3.2 we will see that the curve is approximately a parabola.*
2 The Numerical Approach

This section will formalize the following “story”: First, consider a model of an economy without any imperfections. We call this model the Walrasian Equilibrium Model (WEM). Suppose, we use this model to analyze the economic consequences of immigration. Next, we set up a model that allows for some market imperfections and reevaluate the effects of immigration using this model. Then, we compare the predictions of these two models. If the predictions differ in an essential manner even through the market imperfections are very small, than we should be suspicious about the results of the WEM. In the following, this “story” will be discussed in a general and abstract setting in order to highlight the principal mechanism behind the numerical analysis done in the subsequent sections.

Consider an economy hit by any exogenous shock that is measured by an index $s$. We want to calculate the economic consequences of this shock on a certain variable $W$, e.g. income, welfare, etc. At first we write down an WEM and carry out a comparative analysis, e.g. we calculate $W(s_1) - W(s_0)$. We hope that $W(s_1) - W(s_0)$ gives us an acceptable approximation of the real world effects.

To put this in concrete terms, we perform the following test: We write down equilibria models for the economy $E(\varepsilon)$, where the parameter $\varepsilon$ measures the closeness to the WEM. To offer an example, $\varepsilon$ could be the measure of the market power of firms or bargaining power of trade unions. The parameterization is chosen such that $E(0)$ coincides with the WEM. The general model therefore embeds the perfectly competitive economy. In the next step, we calculate the values of $W(s_1, \varepsilon) - W(s_0, \varepsilon)$, $\varepsilon \geq 0$. If for all $\varepsilon$ sufficiently close to zero the approximation

$$W(s_1, \varepsilon) - W(s_0, \varepsilon) \approx W(s_1, 0) - W(s_0, 0)$$

holds, then the predictions of the WEM can be defended and we call the WEM unproblematic.

The main point is to operationalize the meaning of “the approximation holds”. To achieve this we use a standard technique of Mathematical Numerical Analysis, viz. we study the Taylor expansion of $W(x, \varepsilon)$ with respect to $s$ treating $\varepsilon$ as a parameter:

$$W(s_1, \varepsilon) = W(s_0, \varepsilon) + W'(s_0, \varepsilon)(s_1 - s_0) + \frac{1}{2} W''(s_0, \varepsilon)(s_1 - s_0)^2 + \sum_{k=3}^{\infty} c_k W^{(k)}(s_0, \varepsilon)(s_1 - s_0)^k,$$  \hspace{1cm} (1)

where we assume that $W \in C^\infty$ holds. Assume for a moment $W''(\cdot, \cdot)$ is not zero,
whereas the terms of higher order are zero. From an approximative point of view we can reformulate the problem and ask whether the following two expansions are similar

\[
W(s_1, \varepsilon) - W(s_0, \varepsilon) = W'(s_0, \varepsilon)(s_1 - s_0) + \frac{1}{2} W''(s_0, \varepsilon)(s_1 - s_0)^2, \quad (2)
\]

\[
W(s_1, 0) - W(s_0, 0) = W'(s_0, 0)(s_1 - s_0) + \frac{1}{2} W''(s_0, 0)(s_1 - s_0)^2. \quad (3)
\]

Now the central idea: We compare in a \(\varepsilon\)-neighborhood of zero those non-zero terms of the Taylor expansion given by (2) and (3) that are respectively of the smallest order. Thus, if in the expansions (2) and (3) the coefficients \(W'(s_0, 0)\) and \(W'(s_0, \varepsilon)\) are both strictly positive for all \(\varepsilon\) sufficiently close to zero then the WEM is unproblematic. If however \(W'(s_0, 0) = 0\) and for all \(\varepsilon\) in a neighborhood of zero \(W'(s_0, \varepsilon) > 0\) holds, then we consider the WEM as problematic: The effect captured by the WEM, i.e. the effect captured by \(\frac{1}{2} W''(s_0, 0)\), is the numerically less important effect (second-order) in the extended model. If reality differs from the walrasian case, we would stress other effects.

In this sense the WEM is structurally unstable.

To develop the idea for the general case, we define \(\alpha(W(s, \varepsilon)) := \min\{i \in \mathbb{N} : c_i(s, \varepsilon) \neq 0\}\). The coefficient \(c_{\alpha(W(s, \varepsilon))}\) will be called the leading coefficient of \(W(s, \varepsilon)\). Furthermore, for two real numbers \(x, y \in \mathbb{R}\) we say that they have the same signature in the strict sense if and only if both are either strictly positive or both are strictly negative.

**Definition:** The Walrasian Equilibrium Model (WEM) is called unproblematic if and only if there exists a \(\varepsilon_0 > 0\) such that for all \(\varepsilon < \varepsilon_0\) the leading coefficients of \(W(s_0, 0)\) and \(W(s_0, \varepsilon)\) are of the same order and have the same signature in the strict sense.

**Remark 1:** We must keep in mind that if \(\varepsilon\) approaches zero the coefficients of the Taylor expansion of \(W(s_0, \varepsilon)\) will presumably converge to those of \(W(s_0, 0)\). Therefore, we cannot take this as fact that the WEM is problematic in the sense of the definition, as non-questionable evidence that the WEM is useless. Maybe the economy’s \(\varepsilon\) is very close to zero and/or the value \(s_1\) too far from \(s_0\) to take the Taylor approximation seriously. However, sceptism is appropriate if a WEM is problematic.

**Remark 2:** The distinction between first-order and second-order effects is well known in economics. Akerlof and Yellen (1985a, 1985b) provide a related but different approach. They show that small reasons may have large consequences. However, they fix the structure of the model (\(\varepsilon\) in our notation). In this paper it is argued that a shock may have
first and second-order effects and that the WEM may emphasize the small and ignore the large effects. What varies is the structure (the $\varepsilon$) of the model not the size of the shock.

3 The Numerical Approach Applied to Immigration

3.1 Immigration Surplus Function

Full Employment

The economy we are considering produces one good with $N$ factors of production. The production function $F(L_1, \ldots, L_N)$ is neoclassical. The factor prices are given by their marginal products and for the moment we assume full employment. Consider the effect of immigration of the first factor on the income of the “native” factors. We define the immigration surplus function:

$$Q(\Delta L_1) := F(\overline{L}_1 + \Delta L_1, \ldots) - \frac{\partial F(\overline{L}_1 + \Delta L_1, \ldots)}{\partial L_1} \Delta L_1 - F(\overline{L}_1, \ldots),$$

where $\overline{L}_1$ denotes the native factors, i.e. factor owned by natives. The first term on the right hand is aggregate income after immigration, the second term equals the income of the immigrated factor and the last term is aggregate income before immigration. Thus $Q$ equals the change in aggregate income that natives experience because immigration took place. In the next step we calculate the Taylor expansion of $Q$ in order to relate the analysis to the framework of the proceeding section:

$$Q(\Delta L_1) = Q(0) + 0 \cdot \Delta L_1 - \frac{1}{2} \frac{\partial^2 F}{\partial L_1^2} \cdot (\Delta L_1)^2 + \ldots$$

since $Q'(\Delta L_1) = \Delta L_1 \cdot (\partial^2 F/\partial L_1^2)$. Therefore, the incumbent factors benefit (in aggregate) from immigration, but we can draw the following conclusion.

**Conclusion:** At the margin there are no first-order effects. Immigration – not necessarily of labor – generates only second-order effects.

Allowing for Imperfections

In the following we consider labor migration and fix (and ignore) other factors in all equations. We allow for imperfections. Employment $N(\Delta L)$ is a function of the size of immigration with $N'(\Delta L) > 0$. With full employment we have $N = L + \Delta L$ and therefore $N' = 1$. However, with an imperfection in the labor market a unit increase of labor supply will not lead to a unit increase in employment but $N' < 1$. Employment $N(\cdot)$
is considered as the equilibrium outcome of some kind of wage setting—labor demand process. Several frameworks are possible: Wage setting by trade unions or an efficiency wage model are probably the most popular ones. Such models can be reduced to two equations

\[ P = (1 + \mu)(W/F'((1 - u)L)), \quad (4) \]

\[ W = Pg(u). \quad (5) \]

This system fixes the equilibrium unemployment rate \( u \) and the real wage \( w = W/P \).

A higher unemployment rate induces a lower wage claim \( g'(.) < 0 \). With capital fixed, the equilibrium unemployment will increase with \( L \). Therefore, the unemployment rate is higher after immigration.

Suppose firms operate on their ordinary labor demand curve, i.e. we have \( \frac{\partial F}{\partial L}(N) = W \) or \( \mu = 0 \). Immigration surplus is

\[ Q(\Delta L) = F(N(\Delta L)) - w(\Delta L) \cdot N_I(\Delta L) - F(N(0)). \]

The first derivative of \( Q \) with respect to \( \Delta L \) is

\[ Q'(\Delta L) = F'(N(\Delta L)) \cdot N'(\Delta L) - w(\Delta L) \cdot N'_I(\Delta L) - W'(\Delta L) \cdot N_I(\Delta L) \]

\[ = F'(N(\Delta L))(N'(\Delta L) - N'_I(\Delta L)) - w'(\Delta L) \cdot N_I(\Delta L). \]

The Taylor expansion at zero becomes

\[ Q(\Delta L) = Q(0) + F'(N(0)) \underbrace{(N'(0) - N'_I(0))}_{=0?} \Delta L + o((\Delta L)^2) \]

with a non-zero first-order effect if and only if \( N'(0) - N'_I(0) \neq 0 \) holds.

The difference \( N(\Delta L) - N_I(\Delta L) \) equals the (job-) crowding out of natives. Since the equilibrium unemployment rate rises with labor supply, it follows that there is crowding out and therefore \( N'(0) - N'_I(0) < 0 \). We conclude that there is a negative first-order effect. Suppose that crowding out is proportional to the size of immigration:

\[ N(\Delta L) - N_I(\Delta L) = -\varepsilon \Delta L. \]

We can rewrite the Taylor expansion at zero:

\[ Q(\Delta L) = Q(0) - \varepsilon F'(N(0)) \Delta L + o((\Delta L)^2). \]

Referring to the definition of section 2 we can conclude that the WEM is problematic.
Next, we allow for an imperfection in the goods market. In this case $w = \frac{1}{1+\mu} F'$ and the Taylor expansion becomes

$$Q(\Delta L) = Q(0) + F'(N(0)) \left( N'(0) - \frac{N_0'(0)}{1 - \mu} \right) \Delta L + o((\Delta L)^2).$$

The first-order effect of immigration vanishes if and only if

$$N'(0) = \frac{N_0'(0)}{1 + \mu}.$$ 

Generically, there will be a first-order effect, but we can’t say whether it is positive or negative. In any case, the generically dominate effect will not be the one of Berry and Soligo.

### 3.2 Back-of-the-Envelope Calculations

Because of remark 1 in section 3.1 we can’t be sure whether we discuss a theoretical curiosity or a real quantitative issue. To prove the relevance, we perform back-of-the-envelope calculations in the framework of Berry and Soligo: A closed one-good-economy is hit by a wave of immigration. We study income effects and not welfare (but see section 3.3) because the interpretation of parameters of production functions and demand curves is more straight forward than the measurement of welfare. By assumption, labor is homogenous. The supply side of the economy is given by a neoclassical production function

$$Y = F(L, K)$$

with constant returns to scale and positive but decreasing marginal productivities. We assume that all factors are inelastically supplied and that their markets clear. In equilibrium the factor prices are equal to their respective marginal products. The question is: “What is the effect of immigration on the income of the natives?” Because of

$$Q' = -F''(L + \Delta L, K), (\Delta L > 0),$$

the difference between the income of the natives before and after the wave of immigration is positive:

$$Q(M) := -\int_0^M F''(L + \lambda, K) \lambda d\lambda > 0. \tag{6}$$

This is a wonderful result which can be generalized to a very large extent. The bottom line is: As long as the migrants differ from the natives, the natives are benefiting from immigration (e.g. Wong (1995)).

Yet, there is a caveat that was first addressed by Razin and Sadka (1995a, 1995b). We consider a very (!) simplified version of their model. So far we have assumed that wages
are flexible: the wage rate is lower after immigration and labor is again fully employed. Let us – following Razin and Sadka – assume that wages are completely inflexible. Again we consider the effects on the income of the natives assuming that migrants and natives are equally successful on the labor market so that their respective employment rates are the same. Then the immigration surplus is \( Q(\Delta L) = \bar{Y} - \pi e \Delta L - \bar{Y} = -m \bar{\pi} \bar{T} \), which is of course negative. Due to the rigidity, aggregate income is fixed and as long as the migrants receive anything the natives will lose. However – and this was stressed by Razin and Sadka – there is a quantitative nuance. Razin and Sadka demonstrated by simulation (for a more complex economy) that the economic consequences of migration are large relative to those in the full employment case.

What drives the result can be sketched as follows. In the flexible case the per head income effect is given by

\[
\frac{Q}{Y} = 0.5 \alpha |\varepsilon_{wL}| \alpha^2,
\]

where \( m \) is the share of immigrants to total population, \( \alpha \) is the wage share and \( \varepsilon_{wL} \) is the elasticity of the wage with respect to employment.\(^1\) If we consider the corresponding expression for the rigid case

\[
\frac{Q}{Y} = -\alpha m
\]

we note the major difference. The economic consequences are of second-order in the case of full employment whereas they are of first-order in the rigid case. To provide a numerical example consider table 1. It shows a back of the envelope calculation, which assumes that the shares of wages is 0.7 and the elasticity of the wage with respect to employment is -1 (e.g. Hamermesh (1993)).

<table>
<thead>
<tr>
<th>( m )</th>
<th>0.005</th>
<th>0.01</th>
<th>0.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{flex}}{%} )</td>
<td>0.000875</td>
<td>0.0035</td>
<td>0.014</td>
</tr>
<tr>
<td>( \frac{\text{fix}}{%} )</td>
<td>-0.35</td>
<td>-0.7</td>
<td>-1.4</td>
</tr>
<tr>
<td>( \frac{\text{fix/flex}}{%} )</td>
<td>400</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 1: Back-of-The-Envelope I

The first two rows give the difference of income in percent. The third row gives the quotient of the respective value of the the second and the first row.

Of course, neither of the scenarios above are realistic, however the comparison triggers the suspicion that the WEM is highly misleading. Above we assumed complete rigidity and obtained large negative income effects. However, complete rigidity is quite extreme,

\(^1\)See Borjas (1995). The parabola can be obtained from equation (6).
possibly some flexibility is sufficient to restore a positive immigration surplus. In fact, there exists a certain degree of flexibility where the immigration surplus is zero. For complete flexibility the income effects are positive and for complete rigidity they are negative. With an intermediate value argument there is a degree of flexibility that is sufficient to generate a zero income effect. If the attack on the WEM is justified then the critical degree of flexibility is very close to complete flexibility. To show this we consider the immigration surplus

\[ Q = F(N_d(w_1)) - w_1(1 - u)\Delta L - F(N_d(w_0)) \]

where \( w_0 \) and \( w_1 \) denote the wages before and after immigration, \( N_d \) denotes labor demand and \( u \) the unemployment rate. As before, we assume that migrants and natives are equally successful on the labor market. The condition for \( Q = 0 \) is \( F(N_d(w_0))/F(N_d(w_1)) = 1 - \alpha m \). For \( m = 0.03 \) (\( m = 0.05 \)) and \( \alpha = 0.7 \) this implies an increase of output by 2.15 % (3.626 %). This output increase in turn implies via the output-elasticity of labor and the wage-elasticity of employment an implied decline of wage of \( \hat{w} = (\varepsilon_{wN}/0.7)\hat{Y} \).

The numerical values are given in Table 2. The numbers in parenthesis are the decline of wages triggered by immigration in the case of full employment.

<table>
<thead>
<tr>
<th>( \varepsilon_{wN} )</th>
<th>0.3 %</th>
<th>0.7 %</th>
<th>1.3 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m=3% )</td>
<td>0.921 (0.928) %</td>
<td>2.15 (2.17) %</td>
<td>4.00 (4.02) %</td>
</tr>
<tr>
<td>( m=5% )</td>
<td>1.554 (1.59) %</td>
<td>3.626 (3.68) %</td>
<td>6.734 (6.84) %</td>
</tr>
</tbody>
</table>

Table 2: Back-of-The-Envelope II

The table shows the implied wage change. The numbers in parenthesis are the corresponding full employment numbers.

We conclude that the necessary flexibility is almost complete flexibility. In other words, already some inflexibility will be sufficient to make the result of the WEM misleading.

### 3.3 Ostroy’s Theorem and Immigration

All arguments made so far relate to effects to income. We focus on income and sketch only welfare because the issue can be quantified more easily with income than with utility. However, because of Ostroy’s Theorem (1980) they can be generalized to welfare as well (see Mas-Colell (1980, section 7.2 and 7.6)). This theorem states that in a Walrasian Equilibrium every agent receives his marginal contribution to the economy (see Mas-
Colell (1980, section 7.2) for the definition “contribution”). It is evident that at the margin immigrants do not alter the welfare of the natives, since the additional person receives exactly his contribution therefore leaving everybody else’s welfare unchanged (at least in the first-order approximation). As a consequence, the effects that show up in the WEM are effects of second-order only.

4 Conclusion

When thinking about the economic consequences of immigration it is convenient to use the Berry-Soligo result and its generalization: with competitive markets natives gain from immigration because of improved gains from exchange. This paper shows that this analysis is structurally unstable: Small imperfections make the Berry-Soligo effects negligible quantities. With back-of-the-envelope calculations the problem is shown numerically. Via the Taylor expansions the theoretical background is documented. The major arguments are made for income effects. However, because of Ostroy’s Theorem they are true for utility as well.

One may claim: “Ignoring imperfection is justified if they are small.” This paper shows that this logic does not apply in the case of immigration. Even if market frictions are small the effects found in the frictionless case are a bad approximation because of the structural instability discussed here.

References

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