Economic Consequences of the Posted Workers Directive

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Abstract

This paper analyzes the consequences of an increase of the minimum wage for foreign workers in the construction sector, being implied by the EU Posted Workers Directive. Due to the rising price of construction services, the factor demand for both construction services and capital in the tradeable good sector falls, and the wage rate in this sector declines. While the share of domestic workers increases with respect to both foreign workers and capital in the construction sector, this need not suffice to reduce unemployment. A possible higher level of employment of natives is not sufficient to raise natives' welfare.

JEL classification: F20, J38, J61, J68
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1 Introduction

At the end of 1996 the European Union adopted the Posted Workers Directive. The main content of this directive is that any labor contract between a firm and a worker who is sent temporarily abroad within the EU is in many respects governed by the labor market laws in the host country, in particular by its minimum wage rulings (for details see Davies (1997)). In Germany, the parliament had accepted a similar Posted Workers Law for workers in the construction sector in the same year. The initial situation was that between 100,000 and 200,000 posted workers from Portugal, Ireland and the United Kingdom were employed in the German construction sector (Eekhoff (1996), Noll (1996)), while at the same time about 300,000 German construction workers were unemployed (Noll (1996), Straubhaar (1996a,b)). The wages of the posted workers lay between 25 % and 34 % below the levels of their German counterparts (Eekhoff (1996)).

Since Germany has no general minimum wage, trade unions and employers in the construction sector introduced a new low-wage group of workers in their wage contract. The corresponding wage was declared binding for all workers and firms in this sector. While the Posted Workers Directive might aim at improving the conditions of the posted workers (Smith and Villiers (1996)), the German Posted Workers Law obviously was enacted in order to protect German workers in the construction sector against competition from abroad. Noting that the directive will mainly be applied in the construction sector, the economic consequences of the two products of legislation are obviously the same.

This paper explores the consequences of the directive in the host country. The increase in the minimum wage of foreign workers is effective and is not neutralized by side payments between employers and posted workers (Straubhaar (1996a), Rötte and Zimmermann (1998)). We consider the impacts on employment of both native and posted workers, the wage rate in the non-construction part of the economy, relative prices, the allocation of capital and the welfare of both the natives and the foreign workers.

The paper contributes to the literature on the consequences of migration in the presence of unemployment in the receiving country, which is still very limited. Notable exceptions are Rivera-Batiz (1981), Brecher and Choudri (1987), Djajić (1993), Schmidt et al. (1994), and Razin and Sadka (1995). However, none of these models seems appropriate when dealing with the is-
sues raised by the Posted Workers Directive. The analysis combines the view of a small open economy with non-traded goods (Rivera-Batiz (1982)) with the idea that migrants and natives are imperfect substitutes in production (Ethier (1985)). Moreover, it is taken into account that the good produced in the construction sector plays a double role as consumption good and production input.

A two-sector model of a small open economy is considered. Posted workers are only found in the sector that produces a good which is not traded internationally. In this sector, being interpreted as the construction sector, the wages of both home and foreign workers are fixed. The non-traded good is not only consumed, but also serves as an input for the tradeable good. The consequences of an increase in the posted workers’ wage are analyzed.

The increase in their wage raises the unit cost of the non-traded good which implies a higher price. This yields a decrease in factor demand of the traded good sector. Moreover, the wages of the workers in this sector fall, and capital is transferred abroad. In the non-traded good sector, less foreigners will be employed. Production is restructured in favor of domestic labor, while the ratio between capital and domestic labor decreases.

Even if employment of natives rises, this does not ensure that the natives’ total income increases. In addition, even if the natives’ income increases, their welfare may well decline due to the higher price of the non-traded good. It is shown that the impacts on the welfare of the natives and the wage bill of the posted workers are ambiguous.

2 The model

Two goods are produced in the economy. The first sector produces a traded good which is the numeraire, while the second good, construction services, is not tradeable. Posted workers are only employed in the second sector. The first sector produces its output $X_1$ with capital $K_1$, labor $L_1$ and construction services $X_2^1$. The production function $X_1 = X_1(K_1, L_1, X_2^1)$ is strictly increasing in each argument, strictly quasi-concave, and exhibits constant returns to scale. All pairs of factors are q-complements (see Bond (1989)), i.e. all cross derivatives of the production function satisfy $X_{1ij} > 0$ for $i \neq j$, $i,j \in \{ K_1, L_1, X_2^1 \}$. Profits in the first sector amount to

$$\pi_1 = X_1(K_1, L_1, X_2^1) - rK_1 - w_1L_1 - P_2X_2^1$$
where \( r, w_1 \) and \( P_2 \) denote the interest rate, the wage rate in the first sector, and the price of the second good, respectively. Due to its linear homogeneity, production per worker in this sector is \( \frac{X_1}{L_1} = f(k_1, z_1) \) with \( k_1 = \frac{K_1}{L_1} \) and \( z_1 = \frac{X_1}{L_1} \). Factors are paid their respective marginal productivities:

\[
\begin{align*}
    f_1(k_1, z_1) - r &= 0, \\
    f_2(k_1, z_1) - P_2 &= 0, \\
    f(k_1, z_1) - k_1 f_1(k_1, z_1) - z_1 f_2(k_1, z_1) - w_1 &= 0,
\end{align*}
\]

with \( f_1 := \frac{\partial f}{\partial k_1} \) and \( f_2 := \frac{\partial f}{\partial z_1} \). Since \( F \) is linear homogeneous and strictly quasi-concave, the function \( f \) is strictly concave. The construction sector produces its output \( X_2 \) with capital \( K_2 \), home labor \( L_2 \), and foreign labor \( L_F \) under constant returns to scale with positive marginal productivities and a strictly quasi-concave production function. Profits in the second sector are

\[
\pi_2 = P_2 X_2(K_2, L_2, L_F) - r K_2 - w_2 L_2 - w_F L_F,
\]

where \( w_2 \) and \( w_F \) respectively denote the fixed wage rates of the domestic and foreign workers. The wage rate of foreign workers falls short of the wage rate of native workers in the construction sector, i.e. \( w_2 > w_F \). Native workers are neither mobile between sectors nor across borders. This assumption has been chosen in order to keep the analysis simple. As an alternative, one could imagine that, as in Harris and Todaro (1970), the native workers migrate between sectors until the expected wages are equalized. While full employment in the first sector prevails, minimum wage unemployment for domestic workers occurs in the construction sector. Production per domestic worker can be written as \( \frac{X_2}{L_2} = g(k_2, z_2) \) with \( k_2 = \frac{K_2}{L_2} \) and \( z_2 = \frac{L_F}{L_2} \). Again applying the marginal productivity rule for factor prices yields

\[
\begin{align*}
    P_2 g_1(k_2, z_2) - r &= 0, \\
    P_2 g_2(k_2, z_2) - w_F &= 0, \\
    P_2 g(k_2, z_2) - P_2 k_2 g_1(k_2, z_2) - P_2 z_2 g_2(k_2, z_2) - w_2 &= 0,
\end{align*}
\]

with \( g_1 := \frac{\partial g}{\partial k_2} \) and \( g_2 := \frac{\partial g}{\partial z_2} \). It may be asked whether a production function exists where the two types of labor can be aggregated, the function
exhibits constant returns to scale with respect to capital and labor, and both domestic and foreign workers will generally be employed. The answer is in the affirmative, as shown by the example

\[ X_2(K_2, L_2, L_F) = Y(K_2, L_2 + q(L_2/L_F)L_F) \]

with \( q \in (0,1) \), \( q' > 0 \) and \( q'' < 0 \). Hence, the productivity of foreign workers falls short of the productivity of their native counterparts, but depends positively on the share of natives within the workforce in the construction sector. This positive impact, which may be justified by easier communication due to the natives’ superiority with respect to command of their language, diminishes with increasing share of the natives. Moreover, the elasticity of \( q \) with respect to \( \frac{L_2}{L_F} \) is required to be less than unity, i.e. \( \frac{q'}{q} < 1 \), in order to ensure that increasing the foreign work force always yields a higher output.

We assume that the cross derivatives of the production function \( X_2 \) satisfy

\[ X_{KL} \equiv \frac{\partial^2 X_2}{\partial K_2 \partial L_2} > 0, \quad X_{KF} \equiv \frac{\partial^2 X_2}{\partial K_2 \partial L_F} > 0, \quad \text{and} \quad X_{LF} \equiv \frac{\partial^2 X_2}{\partial L_2 \partial L_F} < 0. \]

In other words, while foreign and domestic labor constitute \( q \)-substitutes, the two other pairs of factors are \( q \)-complements. Capital is internationally mobile and always fully employed. Every unit of capital earns the fixed world market interest rate \( r \). Hence, the natives’ capital income always amounts to \( rK \), where \( K \) represents the stock of capital owned by natives. Households have identical homothetic preferences. We can therefore consider a representative native household maximizing its utility function \( U(C_1, C_2) \) with \( C_1 \) and \( C_2 \) denoting consumption of the first and the second good, respectively, subject to the budget constraint

\[ \pi_1 + \pi_2 + w_1 L_1 + w_2 L_2 + rK - C_1 - P_2 C_2 = 0. \]

Due to Euler’s theorem, \( \pi_1 = \pi_2 = 0 \) holds.

Optimization leads to Marshallian consumption functions \( C_1(P_2, I) \) and \( C_2(P_2, I) \) with \( I = w_1 L_1 + w_2 L_2 + rK \) representing the income of the natives. We assume that the foreign workers only spend their income on the tradeable good. This reflects that temporary migrants usually exhibit a high savings rate, which can be justified by preferences for consumption in their home country (Djajić and Milbourne (1988)) or just by aiming at smoothing their
intertemporal consumption profile (Galor and Stark (1990)). Since capital owners can only repatriate their income from abroad by means of transferring units of the tradeable good, the market clearing conditions are

\[
L_2g(k_2, z_2) - C_2(P_2, w_1L_1 + w_2L_2 + r\bar{K}) - L_1z_1 = 0, \tag{7}
\]

\[
L_1f(k_1, z_1) + r(\bar{K} - L_1k_1 - L_2k_2) - C_1(P_2, w_1L_1 + w_2L_2 + r\bar{K}) - w_Fz_2L_2 = 0. \tag{8}
\]

The equations (1)-(6) imply

\[
L_1f(k_1, z_1) = rk_1L_1 + w_1L_1 + P_2z_1L_1, \tag{9}
\]

\[
L_2P_2g(k_2, z_2) = rk_2L_2 + w_2L_2 + w_Fz_2L_2. \tag{10}
\]

Adding up these two equations, inserting for \(L_2g(k_2, z_2)\) from the market clearing condition (7), and rearranging yields

\[
L_1f(k_1, z_1) + r(\bar{K} - L_1k_1 - L_2k_2) - w_Fz_2L_2 - [r\bar{K} + w_2L_2 + w_1L_1 - P_2C_2(P_2, w_1L_1 + w_2L_2 + r\bar{K})] = 0. \tag{11}
\]

Since \(C_1(P_2, l) = I - P_2C_2(P_2, l)\), this is just the second market clearing condition (8), which can therefore be omitted. Hence, the seven equations (1)-(7) determine the seven endogenous variables \(k_1, z_1, w_1, k_2, z_2, P_2, L_2\).

3 Comparative statics

This section discusses the impacts of an increase in the foreign workers' wage rate \(w_F\). We assume that an interior solution exists which satisfies (1)-(7), and that this solution is unique.

The Jacobian of the system of equations (1)-(7) is

\[
A = \begin{bmatrix}
  f_{11} & f_{12} & 0 & 0 & 0 & 0 & 0 \\
  f_{21} & f_{22} & 0 & 0 & 0 & -1 & 0 \\
  a_{31} & a_{32} & -1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & P_{2g11} & P_{2g12} & g_1 & 0 \\
  0 & 0 & 0 & P_{2g21} & P_{2g22} & g_2 & 0 \\
  0 & 0 & 0 & a_{64} & a_{65} & a_{66} & 0 \\
  0 & -L_1 & -C_2L_1 & L_{2g1} & L_{2g2} & -C_2P & g(k_2, z_2) - w_2C_2l
\end{bmatrix}
\]
where
\[
\begin{align*}
a_{31} &= -k_1 f_{11} - z_1 f_{21}, \\
a_{32} &= -k_1 f_{12} - z_1 f_{22}, \\
a_{64} &= -P_2[k_2 g_{11} + z_2 g_{21}], \\
a_{65} &= -P_2[k_2 g_{12} + z_2 g_{22}], \\
a_{66} &= g(k_2, z_2) - k_2 g_1(k_2, z_2) - z_2 g_2(k_2, z_2).
\end{align*}
\]

Note that \( f_1 = X_{1K}, f_{11} = X_{1KK} L_1, f_{12} = X_{1KX} L_1, f_{11} k_1 + f_{12} z_1 = X_{1KK} K_1 + X_{1KX} X_{2} = -X_{1KL} L_1 \). The last equality is implied by the rule that derivatives of a linear homogeneous function are homogeneous of degree zero. The derivatives of \( g \) can be transformed in an analogous fashion.

Evaluating the determinant of the Jacobian yields
\[
\Delta = -[g(k_2, z_2) - w_2 C_{2f}] P_2^2 \det \begin{bmatrix} g_{11} & g_{12} & g_1 \\ g_{21} & g_{22} & g_2 \\ g_{a64} & g_{a65} & a_{66} \end{bmatrix} \begin{bmatrix} f_{11} f_{22} - f_{21} f_{12} \end{bmatrix},
\]

where
\[
\begin{align*}
\det \begin{bmatrix} g_{11} & g_{12} & g_1 \\ g_{21} & g_{22} & g_2 \\ g_{a64} & g_{a65} & a_{66} \end{bmatrix} &= [g(k_2, z_2) - k_2 g_1 - z_2 g_2][g_{11} g_{22} - g_{12} g_{21}] \\
&
- [k_2 g_{11} + z_2 g_{21}][g_{12} g_{22} - g_{11} g_{22}] \\
&
+ [k_2 g_{12} + z_2 g_{22}][g_{11} g_{22} - g_{12} g_{21}] \\
&= g(k_2, z_2)[g_{11} g_{22} - g_{12} g_{21}].
\end{align*}
\]

Now
\[
\begin{align*}
g_{11} g_{22} - g_{12} g_{21} &= L_2^2 [X_{2KK} X_{2FF} - X_{2FK} X_{2KF}] > 0, \\
f_{11} f_{22} - f_{12} f_{21} &= L_1^2 [X_{1KK} X_{1XX} - X_{1KX} X_{1XX}] > 0
\end{align*}
\]

hold due to the strict concavity of the functions \( f \) and \( g \). Moreover, \( g(k_2, z_2) - w_2 C_{2f} \) is positive due to \( g(k_2, z_2) - g_1 k_1 - g_2 z_2 - \frac{w_2}{P_2} = 0 \) and \( P_2 C_{2f} = 1 - C_{1f} < 1 \). This suffices to ensure that \( \Delta < 0 \).

Proposition 1 deals with the impact of an increase in the posted workers' wage on the price of the non-traded good and the production structure in the traded good sector.
Proposition 1  An increase in the posted workers’ wage rate raises the price of the non-traded good but lowers the wage rate in the traded good sector. Both the capital-labor ratio and the input of the non-traded good in this sector decrease.

Proof: The vector of partial derivatives of (1)-(7) with respect to $w_F$ is

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$  

The implicit function theorem then implies

$$\frac{dk_1}{dw_F} = -\frac{\Delta_{k_1w_F}}{\Delta},$$  

$$\frac{dz_1}{dw_F} = -\frac{\Delta_{z_1w_F}}{\Delta},$$  

$$\frac{dw_1}{dw_F} = -\frac{\Delta_{w_1w_F}}{\Delta},$$  

$$\frac{dP_2}{dw_F} = -\frac{\Delta_{P_2w_F}}{\Delta},$$

where $\Delta_{xw_F}$ with $x \in \{k_1, z_1, w_1, P_2\}$ denotes the determinant of the matrix that arises if in matrix A the column vector of partial derivatives of (1)-(7) with respect to $x$ is replaced by vector $b$.

It turns out that

$$\Delta_{k_1w_F} = [g(k_2, z_2) - w_2 C_{21}] f_{12} P_2 \text{det} \begin{bmatrix} g_{11} & g_{12} \\ a_{64} & a_{65} \end{bmatrix} < 0,$$

$$\Delta_{z_1w_F} = -[g(k_2, z_2) - w_2 C_{21}] f_{11} P_2 \text{det} \begin{bmatrix} g_{11} & g_{12} \\ a_{64} & a_{65} \end{bmatrix} < 0,$$

$$\Delta_{P_2w_F} = -[g(k_2, z_2) - w_2 C_{21}] [f_{11} f_{22} - f_{12} f_{21}] P_2 \text{det} \begin{bmatrix} g_{11} & g_{12} \\ a_{64} & a_{65} \end{bmatrix} > 0,$$

$$\Delta_{w_1w_F} = -[g(k_2, z_2) - w_2 C_{21}] [f_{11} a_{32} - f_{12} a_{31}] P_2 \text{det} \begin{bmatrix} g_{11} & g_{12} \\ a_{64} & a_{65} \end{bmatrix} < 0.$$
since

\[ f_{11}a_{32} - f_{12}a_{31} = z_1[ f_{11}f_{12} - f_{11}f_{22} ] < 0, \]

\[ g_{11}a_{65} - g_{12}a_{64} = P_2 z_2 [ g_{12}g_{12} - g_{11}g_{22} ] < 0 \]

hold due to the strict concavity of the functions \( f \) and \( g \).

Increasing the foreign workers' wage rate yields a higher unit cost of the non-traded good which implies a higher product price. It then follows that in the traded good sector the marginal productivity of the construction input falls short of its price. Thus, the factor demand for the non-traded good will be reduced. Since all factors in the traded good sector are q-complements, the lower input of the non-traded good decreases the marginal productivities of capital and labor. Therefore, capital will be transferred abroad, while the wage rate in this sector will fall.

Proposition 2 states that the structure of production in the non-traded good sector changes in favor of native labor.

**Proposition 2** Increasing the wage rate of the posted workers lowers both the share of foreign workers in the non-traded good sector and their number. The ratio between capital and domestic labor in this sector decreases.

**Proof:** The implicit function theorem implies

\[
\begin{align*}
\text{sgn} \left[ \frac{dk_2}{dw_F} \right] &= \text{sgn}[\Delta_{k_2w_F}], \\
\text{sgn} \left[ \frac{dz_2}{dw_F} \right] &= \text{sgn}[\Delta_{z_2w_F}], \\
\text{sgn} \left[ \frac{dL_F}{dw_F} \right] &= \text{sgn}[L_2 \Delta_{z_2w_F} + z_2 \Delta_{L_2w_F}],
\end{align*}
\]

where \( \Delta_{xw_F} \) with \( x \in \{k_2, z_2, L_2\} \) denotes the determinant of the matrix that arises if in matrix A the column vector of partial derivatives of (1)-(7) with respect to \( x \) is replaced by vector \( b \).

It turns out that

\[
\begin{align*}
\Delta_{k_2w_F} &= -[g(k_2, z_2) - w_2 C_{21}][f_{11}f_{22} - f_{11}f_{21}] \text{det} \left[ \begin{array}{cc} P_2 g_{12} & g_1 \\
& a_{65} & a_{66} \end{array} \right], \\
\Delta_{z_2w_F} &= [g(k_2, z_2) - w_2 C_{21}][f_{11}f_{22} - f_{11}f_{21}] \text{det} \left[ \begin{array}{cc} P_2 g_{11} & g_1 \\
& a_{64} & a_{66} \end{array} \right],
\end{align*}
\]
\[
\Delta_{L_2w_F} = -\det \begin{bmatrix}
  f_{11} & f_{12} & 0 & 0 & 0 & 0 \\
  f_{21} & f_{22} & 0 & 0 & 0 & -1 \\
  a_{31} & a_{32} & -1 & 0 & 0 & 0 \\
  0 & 0 & 0 & P_{2g_{11}} & P_{2g_{12}} & g_1 \\
  0 & 0 & 0 & a_{64} & a_{65} & a_{66} \\
  0 & -L_1 & -C_{21}L_1 & L_{2g_1} & L_{2g_2} & -C_{2P} 
\end{bmatrix}.
\]

Evaluating the determinants yields
\[
P_{2g_{11}}a_{66} - g_1a_{64} = P_2\left[g_{11}[g(k_2, z_2) - z_2g_2] + g_1z_2g_{21}\right] = P_2\left[g_{11}\frac{w_2}{P_2} + g_1[k_2g_{11} + z_2g_{21}]\right] < 0,
\]
\[
P_{2g_{12}}a_{66} - g_1a_{65} = P_2\left[g_{12}[g(k_2, z_2) - z_2g_2] + z_2g_{12}g_{22}\right] = P_2\left[g_{12}\frac{w_2}{P_2} + g_1[k_2g_{22} + z_2g_{12}]\right] > 0,
\]

since \(g_{11} = L_2X_{2KK} < 0, g_{12} = L_2X_{2KF} > 0, g_{11}k_2 + g_{21}z_2 = -L_2X_{2KL} < 0,\) and \(g_{12}k_2 + g_{22}z_2 = -L_2X_{2LF} > 0.\) Noting that \(f_{11}f_{22} - f_{12}f_{21} > 0\) and \(g(k_2, z_2) - w_2C_{21} > 0,\) this shows that \(\frac{dk_2}{dw_F} < 0\) and \(\frac{dz_2}{dw_F} < 0.\)

Furthermore,
\[
+ [f_{12}f_{21} - f_{11}f_{22}] \frac{f_{11}}{a_{32}} C_{21} L_1 - L_1 + f_{12}a_{31} C_{21} L_1] P_2[g_{11}a_{65} - g_{12}a_{64}] = [f_{11}f_{22} - f_{12}f_{21}] \left[ -C_{2P} P_2^2 z_2 [g_{21}g_{12} - g_{11}g_{22}] \right.
\]
\[
+ L_2 P_2 \left[ g_1 \left[ g_{12} \frac{w_2}{P_2} + g_1[k_2g_{12} + z_2g_{21}] \right] - g_2 \left[ g_{11} \frac{w_2}{P_2} + g_1[k_2g_{11} + z_2g_{21}] \right] \right]
\]
\[
+ L_1 \left[ C_{21} z_1 [f_{11}f_{22} - f_{12}f_{21}] - f_{11} \right] P_2^2 z_2 [g_{12}g_{21} - g_{11}g_{22}],
\]
can be derived from
\[
\begin{vmatrix}
    P_{2g11} & P_{2g12} & g_1 \\
    a_{64} & a_{65} & a_{66} \\
    L_{2g1} & L_{2g2} & -C_2P
\end{vmatrix} = -C_2P P_2 [g_{11} a_{65} - g_{12} a_{64}] \\
+ L_{2g1} [P_{2g12} a_{66} - a_{65} g_1] \\
- L_{2g2} [P_{2g11} a_{66} - a_{64} g_1] \\
= -C_2P P_2^2 z_2 [g_{21} g_{12} - g_{11} g_{22}] \\
+ L_2 P_2 \left[ g_1 \left[ g_{12} [g(k_2, z_2) - k_2 g_1 - z_2 g_2] \\
+ g_1 [k_2 g_{12} + z_2 g_{22}] \\
- g_2 [g_{11} [g(k_2, z_2) - k_2 g_1 - z_2 g_2] \\
+ g_1 [k_2 g_{11} + z_2 g_{21}]] \right] \right] \\
= -C_2P P_2^2 z_2 [g_{21} g_{12} - g_{11} g_{22}] \\
+ L_2 P_2 \left[ g_1 \left[ g_{12} \frac{w_2}{P_2} + g_1 [k_2 g_{12} + z_2 g_{22}] \\
- g_2 [g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{21}]] \right] \right].
\]
\[
f_{12} a_{31} C_{2I} - f_{11} [a_{32} C_{2I} + 1] = C_{2I} z_1 [f_{11} f_{22} - f_{12} f_{21} - f_{11}] > 0.
\]

Inserting the results from above yields
\[
L_2 \Delta z_{2wp} + z_2 \Delta L_{2wp} = L_2 [g(k_2, z_2) - w_2 C_{2I}] [f_{11} f_{22} - f_{12} f_{21}] \\
* P_2 \left[ g_1 \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}] \right] \\
+ z_2 [f_{11} f_{22} - f_{12} f_{21}] \left[ -C_2P P_2^2 z_2 [g_{21} g_{12} - g_{11} g_{22}] \\
+ L_2 P_2 \left[ g_1 \left[ g_{12} \frac{w_2}{P_2} + g_1 [k_2 g_{12} + z_2 g_{22}] \\
- g_2 [g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}]] \right] \right] \right] \\
+ L_1 [C_{2I} z_1 [f_{11} f_{22} - f_{12} f_{21}] - f_{11}] \\
* P_2^2 z_2 [g_{12} g_{21} - g_{11} g_{22}].
\]

Note that \( g(k_2, z_2) - w_2 C_{2I} = g_1 k_2 + g_2 z_2 + \frac{w_2}{P_2} (1 - P_2 C_{2I}) > g_1 k_2 + g_2 z_2 > 0, \)

since \( P_2 C_{2I} = 1 - C_{1I} < 1 \) is a consequence of the budget equation \( I = \)
Moreover,
\[
[g(k_2, z_2) - w_2 C_2] \left[ g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}] \right] \\
+ z_2 \left[ g_{12} \frac{w_2}{P_2} + g_1 [k_2 g_{12} + z_2 g_{22}] \right] \cdot \\
- k_2 \left[ g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}] \right] = \\
[g_1 k_2 + \frac{w_2}{P_2} [1 - P_2 C_2]] \left[ g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}] \right] \\
+ z_2 g_1 [k_2 g_{11} + z_2 g_{12}] = \\
\frac{w_2}{P_2} [1 - P_2 C_2] \left[ g_{11} \frac{w_2}{P_2} + g_1 [k_2 g_{11} + z_2 g_{12}] \right] \\
+ z_2 g_1 [k_2 g_{11} + z_2 g_{12}] = \\
+ z_2 g_1 [k_2 g_{11} + z_2 g_{12}] = \\
+ z_2 g_1 [k_2 g_{11} + z_2 g_{12}] < 0
\]

because
\[
k_2 [k_2 g_{11} + z_2 g_{12}] + z_2 [k_2 g_{12} + z_2 g_{22}] = \\
- [K_2 X_2_{KL} + L_F X_2_{FL}] = \\
L_2 X_{2LL} < 0.
\]

Since \( f_{11} f_{22} - f_{12} f_{21} > 0 \), \( g_{11} g_{22} - g_{12} g_{21} > 0 \) and \( f_{11} < 0 \) hold due to the concavity of the production functions \( f \) and \( g \), while \( C_{2P} < 0 \) and \( C_{2I} > 0 \), this suffices to ensure that \( \frac{dL_F}{dw_F} < 0 \).

The foreign workers' marginal productivity falls short of their wage rate since the latter is increasing. This reduces the demand for foreign labor. Consequently, the marginal productivity of capital decreases, while the marginal productivity of domestic labor is positively affected. Therefore, the ratios between domestic and foreign labor, and between domestic labor and capital in the non-traded good sector will increase, while employment of foreign workers will fall.

The change in the number of native workers in the construction sector is ambiguous. This can be explained as follows: The changes in the factor price ratios lead to a change in factor demand in favor of domestic labor. If
the output in the construction sector remains constant, this yields a higher income of the natives, which implies an increased demand for the non-traded good. The increased demand will be satisfied by a proportional increase of all factors. The combined impact of restructuring the production and the induced demand is captured by \([f_{11}f_{22} - f_{21}f_{12}]L_2P_2\left[g_1\left[g_{12}\frac{w_2}{P_2} + g_1[k_2g_{22} + z_2g_{22}]\right] - g_2[g_{11}\frac{w_2}{P_2} + g_1[k_2g_{11} + z_2g_{21}]\right] > 0\). At the same time, consumption will decrease due to the higher price, implying a lower level of production of the non-traded good. This impact is represented by \(C_{2P}P_2^2z_2[f_{11}f_{22} - f_{12}f_{21}][g_{11}g_{22} - g_{12}g_{21}] < 0\). In addition, both the lower input demand of the traded good sector, as shown by \(L_1f_{11}P_2^2z_2[g_{11}g_{22} - g_{12}g_{21}] < 0\), and the lower level of consumption by the workers in that sector, being reflected in \(-L_1z_1C_{21}[f_{11}f_{22} - f_{12}f_{21}][g_{11}g_{22} - g_{12}g_{21}] < 0\), reinforce the tendency towards less production of the non-traded good.

If \(g_{11}g_{22} - g_{12}g_{21}\) is relatively small, the impacts that arise through the increase in the price of the non-traded good and the reactions in the traded good sector will be moderate. In such a setting, unemployment of domestic workers in the construction sector will be reduced. In contrast, if \(f_{11}f_{22} - f_{21}f_{12}\) is close to zero, the substitution effects in the production of the non-traded good will be less important. Since the lower input demand by the tradeable good sector will then constitute the dominating effect, employment of domestic construction workers will decrease.

The change in the production of the non-traded good amounts to

\[
\frac{d[L_2g(k_2, z_2)]}{dw_F} = L_2g_1 \frac{dk_2}{dw_F} + L_2g_2 \frac{dz_2}{dw_F} + g(k_2, z_2) \frac{dL_2}{dw_F}.
\]

Since \(\frac{dk_2}{dw_F} < 0\) and \(\frac{dz_2}{dw_F} < 0\) hold, the employment of domestic labor will increase if the output of the non-traded good does not decline. It turns out that

\[
\text{sgn} \left[\frac{d[L_2g(k_2, z_2)]}{dw_F}\right] = \text{sgn} \left[[f_{11}f_{22} - f_{21}f_{12}]L_2P_2w_2C_{21}
\right.
\begin{align*}
&\left.*\left[g_1\left[g_{12}\frac{w_2}{P_2} + g_1[k_2g_{22} + g_{22}z_2]\right] - g_2[g_{11}\frac{w_2}{P_2} + g_1[k_1k_2 + g_{12}z_2]\right] \right.
\end{align*}
\]
\[
+ g(k_2, z_2)[f_{11}f_{22} - f_{12}f_{21}]C_2P_2^2z_2[g_{11}g_{22} - g_{12}g_{21}]\right]
\]

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+g(k_2, z_2)L_1 \left[ C_{21} z_1 [f_{11} f_{22} - f_{12} f_{21}] - f_{11} \right] \\
\times P_2^2 z_2 [g_{12} g_{21} - g_{11} g_{22}].

Following the same arguments as above, we can see that output in the non-traded good sector will increase if \( g_{11} g_{22} - g_{12} g_{21} \) is relatively small.

The natives' welfare is stated in terms of a representative individual. Hence, we ignore the fact that redistribution occurs in such a way that some natives will definitely lose as a consequence of the Posted Workers Directive. Welfare increases if the winners are able to compensate the losers such that the indirect utility of the representative native increases.

**Proposition 3** Even if the increase of the posted workers' wage rate raises employment of natives in the non-traded good sector, their total income may fall. Even if the income of the natives increases, their welfare can be negatively affected.

**Proof:** The change in income of the natives in terms of the traded good, \( I = w_1 L_1 + w_2 L_2 + r \bar{K} \), is given by

\[
\text{sgn} \left[ \frac{dI}{dw_F} \right] = \text{sgn} \left[ L_1 \frac{dw_1}{dw_F} + w_2 \frac{dL_2}{dw_F} \right].
\]

Since \( \frac{dw_1}{dw_F} < 0 \) by Proposition 1, the first claim is verified.

Let \( v(P_2, I) \) denote the social indirect welfare function. The change in welfare is given by

\[
\frac{dv}{dw_F} = \frac{\partial v}{\partial P_2} \frac{dP_2}{dw_F} + \frac{\partial v}{\partial I} \frac{dI}{dw_F}.
\]

Hence,

\[
\text{sgn} \left[ \frac{dv}{dw_F} \right] = \text{sgn} \left[ \frac{\partial v}{\partial P_2} \frac{dP_2}{dw_F} + \frac{dI}{\partial I} \frac{dI}{dw_F} \right] \]

\[
= \text{sgn} \left[ \frac{dI}{dw_F} - C_2 \frac{dP_2}{dw_F} \right].
\]
according to Roy’s theorem. Noting that $\frac{dP_1}{d\omega_P} > 0$ according to Proposition 1 proves the second claim. □

Proposition 3 contrasts sharply with the analysis of a one-sector model of migration with unemployment in Brecher and Choudri (1987), where the natives will always gain if unemployment of native workers is reduced. In the current model, the immigrants not only displace some native workers, but also contribute to lower cost of production in both sectors and a lower price level. The Posted Workers Directive hurts both the capital owners and the employed workers in the construction sector who have to face a higher price of the non-traded good. The workers in the non-construction sector experience an even higher loss due to the decrease in their wage income. A higher welfare can only be achieved if those natives who gain employment can compensate these losers. Such a situation can arise if $g_{11}g_{22} - g_{12}g_{21}$ is close to zero. The increase in the domestic construction workers’ wage income will then constitute the dominating effect.

The impact of the Posted Workers Directive on the wage bill of the posted workers is ambiguous. On the one hand, their sum of wages may even increase if those who lose employment in the host country receive an income of zero afterwards. This is a simple consequence of the well-known argument that a monopolistic trade union aiming at maximizing the wage bill of its members can set a wage above the market clearing level. On the other hand, given an existing international wage differential, the sum of wages will decrease if all posted workers have to return to their home country.

4 Conclusion

It could be shown that the impacts on the sector which is not directly affected by the Posted Workers Directive are definitely negative. Due to the higher price of the non-traded good, both the factor demand for construction services and the production of the tradeable good decreases, the wage rate declines, and capital is exported. As expected, less posted workers are employed, and the production structure in the non-traded good sector changes in favor of domestic labor.

However, the analysis has not offered an unambiguous answer to the question whether employment of native workers in the construction sector will decrease or increase. Since all natives who do not gain employment will
lose due to the consequences of the increase in the foreign workers' minimum wage, a reduction of unemployment is not sufficient in order to reach a higher level of welfare. The wage bill of the posted workers can either rise or fall. It cannot be excluded that the directive yields a higher level of welfare for both natives and foreigners. Hence, the analysis contributes another argument for possible beneficial impacts of an increase in minimum wages (see the discussion in Dolado et al. (1996)). However, if unemployment is not reduced and all posted workers have to return to their home country, all individuals will lose. In such a setting, politicians clearly should avoid to implement the instruments which make the directive effective.

References