Mortality and Economic Growth

by

Manfred Jäger and Gunter Steinmann
Mortality and Economic Growth

by

Manfred Jäger
Gunter Steinmann

Department of Economics
Martin-Luther-University Halle-Wittenberg
D-06099 Halle (Saale)
January 12, 1997

1We are indebted to Ira Gang and John Komlos for their valuable comments.
1 Introduction

Neoclassical growth theory discusses the consequences of population growth in a very simple manner. Most models treat the rate of population growth as an exogenous variable and emphasize the capital diluting effect of population growth. This leads to the well-known negative assessment of population growth.

The assumption of a given population growth rate is subject to criticism and should be replaced by an endogenous theory of fertility and mortality ([Becker/Barro, 1988]). In addition there are other deficiencies in the analysis of population growth by traditional neoclassical growth theory. We will focus on two points:

(1) Most models do not distinguish between the sources of population growth. However, it certainly matters whether a population growth of 1% stems from a fertility rate of 4% and a mortality rate of 3% or whether it stems from a fertility rate of 2% and a mortality rate of 1%. Countries with high mortality will be subject to faster human capital depreciation than countries with low mortality. As consequence the process of economic growth does not only depend on the growth rate of population but also on the level of the mortality rate ([Steinmann/et al., 1996]).

(2) Traditional models lead to the result that the steady-state growth rate of per capita income is independent of the population growth rate. It is true that a faster population growth reduces both the capital-labour ratio and labour productivity. However, in the long run this process must come to a halt because the increase of capital productivity stimulates the accumulation of capital. The growth rate of capital eventually adjusts to the population growth rate. Therefore traditional neoclassical growth theory concludes that population growth has adverse effects on the level of per capita income but has no impact on the steady-state rate of growth of per capita income. The article of Kelley ([Kelley, 1988]) provides an excellent survey of the literature on the consequences of population growth.

1
Boserup, Simon, Steinmann and others question the relevance of these results. Boserup shows from historical data that societies react to changes in population density by adopting new agricultural technologies ([Boserup, 1981]). Simon and Steinmann present demo-economic growth models in which the rate of technical progress is positively related to the size or the growth of population. A positive link between technical progress and population growth undermines the neoclassical proposition concerning the role of population growth ([Simon/Steinmann, 1984], [Steinmann/Simon, 1980]).

A further step in this direction can be taken using endogenous growth theory. The paper of Kremer provides an example ([Kremer, 1993]). Endogenous growth models draw attention to the impact of physical and human capital for economic growth. We employ the idea of men as "carriers" of human capital to introduce new aspects to population growth theory. The basis of our endogenous growth model is a Rebelo-type economy with a constant returns to scale production function using two reproducible factors, physical and human capital. In addition we relate the depreciation rate of human capital to the mortality rate by taking into account the fact that human capital is embodied in its carrier. The death of a person irrevocably destroys his human capital. Therefore, two countries with identical population growth rates but different levels of mortality and fertility will have different economic growth.

The rest of our paper is organized as follows: In section 2 we present our model. Here, the saving rate is endogenously explained by a Benthamite utility function. In section 3 we replace the Benthamite utility function by a utility function suggested by Barro and Becker and others and discuss the implications. The last section discusses the conclusions of our analysis.
2 The Model

Output $Y$ is produced by physical capital $K$ and human capital $H$. We assume a Cobb-Douglas technology with constant returns to scale:

$$Y = K^\alpha H^{1-\alpha}.$$  \hspace{1cm} (1)

Output can either be consumed or invested in new physical and human capital:

$$Y = C + I_K + I_H.$$ \hspace{1cm} (2)

Gross investment and depreciation determine the growth of physical and human capital. We assume an exogenous depreciation rate for physical capital $\delta$. The mortality rate $m$ is the rate of human depreciation:

$$\dot{K} = I_K - \delta K$$ \hspace{1cm} (3)

and

$$\dot{H} = I_H - m H.$$ \hspace{1cm} (4)

We neglect migration. Therefore the rate of population growth $\dot{L} = n$ is given by the rates of fertility $f$ and mortality:

$$\dot{L} = f - m = n.$$ \hspace{1cm} (5)

With $h = H/L$ and $k = K/L$ we get

$$\dot{k} = i_K - (\delta + n)k,$$  \hspace{1cm} (6)

$$\dot{h} = i_H - (m + n)h,$$ \hspace{1cm} (7)

with $i_H = I_H/L$, $i_K = I_K/L$. Equations (6) and (7) are the fundamental equations of our model.

Since the model does not contain any distortions, we can solve the social planner's problem. We presume a dynastical behaviour with altruistic intergenerational links. In this
section we use a Benthamite utility function, i.e. we assign the same weights to all additional children. Thus we have to maximize

$$\int_0^\infty e^{-(e-n)t}u(c)L(0)dt,$$

s.t.

$$k^\alpha h^{1-\alpha} = c + i_H + i_K,$$

$$\dot{h} = i_H - (m + n)h,$$

$$\dot{k} = i_K - (\delta + n)k,$$

with $c = C/L$. For simplicity we specify $u(c) = \ln c$. The current value Hamiltonian of the optimal control problem is given by

$$H = \ln c + \lambda_h (i_h - (m + n)h) + \lambda_k (i_k - (\delta + n)k),$$

where $i_h$ and $i_k$ are the control variables and $k$ and $h$ are the state variable.

The first-order conditions are

$$\frac{\partial H}{\partial i_h} = \frac{-1}{c} + \lambda_h = 0,$$

$$\frac{\partial H}{\partial i_k} = \frac{-1}{c} + \lambda_k = 0.$$

Thus we have $\lambda_h = \lambda_k$ and therefore $\dot{\lambda}_h = \dot{\lambda}_k$. Regarding the shadow-prices we obtain

$$\dot{\lambda}_h = \frac{\partial H}{\partial h} + (\rho - n)\lambda_h,$$

$$\dot{\lambda}_k = \frac{\partial H}{\partial k} + (\rho - n)\lambda_k,$$

From $\dot{\lambda}_h = \dot{\lambda}_k$ we get

$$MPH - m = MPK - \delta.$$
Equation (19) describes a portfolio of two different kinds of assets. It states that physical and human capital have the same net returns. This condition determines a fixed equilibrium value \( x := (K/H)^* \).

By differentiating \( \frac{1}{c} = \lambda_h = \lambda_k \) with respect to time we obtain

\[
\dot{c} = MPH - m - \rho, \quad (20)
\]
\[
\dot{c} = MPK - \delta - \rho. \quad (21)
\]

We assume that the parameters provide that \( \dot{c} > 0 \) and that the integral in equation (8) converges ([Barro/Sala-i-Martin, 1995], p. 142). Equations (20) and (21) exclude any transitional dynamics of \( \dot{c} \). A deviation of \( K/H \) from its steady-state value \( x \) requires an immediate adjustment to \( x \). This kind of transitional dynamics is very unrealistic. It implies an infinite value of \( i_h \) and \( i_k \) if \( K/H \) deviates from its equilibrium value. To get more realistic out-of-steady-state dynamics, we introduce two restrictions for \( i_h \) and \( i_k \):

\[
i_k \geq 0, \quad i_h \geq 0. \quad (22)
\]

We derive the first order conditions using standard methods ([Feichtinger/Hartl, 1986]):

\[
\frac{1}{c} = \lambda_k + \phi_k, \quad (23)
\]
\[
\frac{1}{c} = \lambda_h + \phi_h, \quad (24)
\]
\[
\phi_k, \phi_h \geq 0, \quad \phi_k i_k = 0 = \phi_h i_h, \quad (25)
\]

where \( \phi_k, \phi_h \) are the Lagrange multipliers associated with the inequalities (22). For an interior solution the differential equations for \( \lambda_h, \lambda_k \) are the same as before. They have to be adjusted if a boundary solution is optimal.

We will restrict our analysis to the specific case \( K/H > x \). In this case \(^2 i_k = 0 \) and \( i_h > 0 \). Therefore \( \phi_h = 0 \). The growth rate of consumption satisfies equation (20) but the marginal productivity is not constant due to changes in \( K/H \). The substitutions \( \chi = c/k \)

\[^2\text{We assume that } m - \delta > 0. \text{ Therefore an inner solution } i_h > 0 \text{ is optimal. Other cases lead to "equivalent" dynamics.}\]
and $\omega = k/h$ allow us to use phase diagram techniques. It can be shown that the following differential equations hold (see appendix):

\[
\dot{\chi} = -\chi^\alpha + \chi\omega + m - \delta, \\
\dot{\chi} = (1 - \alpha)\omega^\alpha - m - \rho + \delta + n.
\]

(26) (27)

For the steady-state conditions, $\dot{\omega} = \dot{\chi} = 0$, we derive

\[
\dot{\omega} = \left(\frac{m + \rho - \delta - n}{1 - \alpha}\right)^{\frac{1}{\alpha}}, \\
\chi = \omega^{(1-\alpha)} - \frac{m - \delta}{\omega}.
\]

(28) (29)

Figure 1: Phase diagram for $K/H \neq x$

In the appendix we show $\dot{\omega} < x$. From figure 1 we see that the economy stays on the saddle-point-path until $\omega$ reaches $x$. At this moment the dynamic regimes changes. The adjustment to the steady-state is completed. Notice that the change of the regime does not lead to a jump of consumption due to the continuity of $c(t)$ ([Feichtinger/Hartl, 1986], Korrelar 6.2).
The steady-state growth path can be transformed into an equivalent AK-model. Substituting $x = K/H$ into the production function gives $Y = AK$ with $A := x^{a-1}$. Barro and Sala-i-Martin demonstrate how to calculate the complete consumption path for an AK-model ([Barro/Sala-i-Martin, 1995], p. 142). We already know the growth rate of $c$. The initial value of the consumption path $c(t_0)$ is given by

$$c(t_0) = (\rho - n)k(t_0),$$

with $t_0$ being the time when $\omega$ reaches $x$.

**Comparative dynamics analysis**

**Increasing Fertility**

We know from equation (19) that the optimal value of $k/h$ is unaffected by fertility. Higher fertility rates force the economy instantly to a lower path of per capita consumption (see equation (30)) but leave the rate of growth unchanged (see (20) and (21)). There are no further transitional dynamics. This result corresponds to the findings of standard neoclassical analysis: Changes in fertility influence the level of per capita consumption but do not alter the growth rate. However the neoclassical arguments for this result differ from our arguments. In the neoclassical world the independence of the steady-state growth rate is caused by an exogenously given rate of technical progress. In our model of endogenous growth the agents can alter the long run growth rate but don’t have any incentive to do so. This is because a higher value of $n$ reduces the effective discount rate $\rho - n$ on the one hand, while at the same time delaying the accumulation of $h$ and $k$ on the other hand. These effects neutralize each other (see equations (20) and (21)).

**Declining mortality**

Faster population growth can be traced to higher fertility or to lower mortality. The neoclassical model produces identical results for both sources of population growth: The long
run growth rate of per capita consumption is affected neither by changes of fertility nor by changes of mortality. Our model produces different results. **Lower mortality leads to higher growth rates of per capita consumption both in the short and in the long run.** Lower mortality rates imply lower values of $x$ (see (19)). It follows from equation (21) that the steady-state growth rate is raised. During the transitional phase the scenario $K/H > x$ applies and we can use our phase diagram. If $m$ declines the curve $\chi = 0$ shifts to the left and the curve $\omega = 0$ moves upwards. During the transitional period $\delta$ decreases with $\omega$ along the saddle-point-path (see equation (20)). The effect on the level of the consumption path remains unclear (see the phase diagram).

The ambiguity of the effect of lower mortality on the level of per capita consumption can be attributed to two counteracting effects: the intertemporal substitution effect and the income effect. On the one hand the higher net returns of the asset $h$ trigger tendencies to substitute today's consumption for future consumption (negative intertemporal substitution effect). On the other hand the higher income allows all generations to consume more (positive income effect).

Faster population growth leads to more capital dilution and weakens the accumulation of human capital per capita. However, while the capital diluting effect is unchecked in the case of increasing fertility, the effect is checked by the lower rate of human capital depreciation in the case of decreasing mortality.

**Declining fertility and mortality**

Our model predicts divergence even between countries with the same population growth rate. Countries with higher $f$ and $m$ face a slower growth of consumption than the countries with lower $f$ and $m$.

After the transition from high fertility and mortality to lower fertility and mortality the developing countries will experience faster growth of per capita consumption even if the population growth rate remains unchanged.
Population policy

The conclusion of our analysis for population policy is obvious. Population policy should place more weight on reducing mortality than on reducing fertility. Lowering mortality is the key factor in improving consumption. Slower population growth is not a necessary precondition of economic growth.

3 Becker/Barro-Type Model

Here we replace the Benthamite utility function with "constant marginal utility of children" by the Becker-Barro utility function with "decreasing marginal utility of children" ([Becker/Barro, 1988]). This Becker-Barro utility function gives less weight to additional children:

\[ \int_0^\infty e^{-\rho t}L(t)^{1-c}u(c)dt, \]  
(31)

with \( \epsilon > 0 \). By maximizing

\[ \int_0^\infty e^{-(\rho-(1-c)n)t}u(c)dt, \]  
(32)

we derive

\[ \hat{c} = MP - \rho - \delta - \epsilon n, \]  
(33)

\[ \hat{c} = MPH - \rho - m - \epsilon n. \]  
(34)

Since the net marginal productivities are identical to those in the Benthamite model, the arbitrage condition remains unchanged.

The comparison of equations (20) and (21) with (33) and (34) makes evident the difference between the Benthamite and the Becker/Barro model. In the Benthamite model an increase in the population growth rate reduces the effective discount rate \( \rho - n \). The fall of the effective discount rate exactly offsets the effects of physical and human capital dilution. This is the reason why the population growth rate does not enter into equations (20) and (21). The replacement of "constant marginal utility of children" by "decreasing marginal
utility of children" destroys the Benthamite balance between the discount rate effect and the capital diluting effect. In the Becker/Barro case faster population growth affects the effective discount rate less than in the Benthamite case. The effective discount rate is only reduced by \(- (1 - \epsilon) \Delta n\). This decrease is too small to neutralize the capital dilution from faster population growth. Therefore the variable \(n\) enters into equations (33) and (34).

**Comparative dynamics analysis**

We assume that the economy has already reached its steady-state equilibrium, i.e. that the value of \(\omega\) corresponds to its equilibrium value given by equation (19).

**Increasing fertility**

An increase of fertility leads in this case to a lower steady-state growth rate of per capita consumption: Since the optimal value of \(k/h\) remains unchanged (this follows from the arbitrage condition (19)) \(\dot{c}\) must decrease according to equation (33). The capital diluting effect is weakened but not completely neutralized by the discount rate effect as it was in the Benthamite model.

**Declining mortality**

The effect of a decline in the mortality rate on the growth rate of per capita consumption depends on the value of \(\epsilon\):

(i) \(\epsilon = 1\) (Mill-type utility function): Equation (34) reduces to \(\dot{c} = MPH - \rho - f\). The decrease of mortality leads to a drop of the growth rate of per capita consumption.

(ii) \(\epsilon = 0\) (Benthamite utility function): Equation (34) coincides with equation (21).

(iii) \(0 < \epsilon < 1\): The consequences of an improved mortality on the growth rate of per capita consumption are ambiguous. For a critical value \(\epsilon = \epsilon^*\) the growth rate of per capita consumption is unaffected by changes of mortality. For \(\epsilon > (\epsilon^*)^*\) the fall of mortality is accompanied by a deterioration (improvement) in the growth rate of per capita consumption.
Declining mortality and fertility

Economic growth accelerates if both mortality and fertility fall in same proportion i.e. if the rate of population growth remains unchanged. This follows from

\[
\frac{d\hat{c}}{df} + \frac{d\hat{c}}{dm} = -(1 - \alpha)\alpha \frac{dx}{dm} < 0. \tag{35}
\]

Equation (35) is proved in the appendix.

4 Conclusion

Both the Benthamite and the Becker/Barro model assume a direct link between mortality and the depreciation of human capital. Therefore, changes in mortality are more important for economic growth than changes in fertility. Economic growth is improved by decreasing fertility as capital dilution diminishes. Decreasing mortality accelerates the accumulation of human capital by reducing the rate of human capital depreciation. Furthermore demographic changes do not only alter the level but also the steady-state growth rates in an endogenous growth model.

This approach provides an alternative explanation of the escape from the malthusian trap, complementing the analysis of [Steinmann et al., 1996] and [Komlos/Artzrouni, 1990]. The escape from the low income trap is driven by both a lower fertility and mortality. This accords well with the historical experience.

Our model carries implications regarding the question of divergence versus convergence of international growth differences. Countries with lower fertility and mortality will experience faster economic growth than countries with higher fertility and mortality. Therefore the sources of population growth are important in assessing the likelihood of long run divergence or convergence.
5 Appendix

Proof of $\dot{\omega} < \dot{x}$: $\dot{\omega} = \left(\frac{\rho - n - \delta + m}{1 - \alpha}\right) \Rightarrow \alpha = \rho - n - \delta + m$.

By assumption: $(1 - \alpha)x^\alpha - \rho - m > 0 \Rightarrow (1 - \alpha)\dot{\omega} = \rho - n - \delta + m < (1 - \alpha)x^\alpha - \delta - n < (1 - \alpha)x^\alpha$. This implies $\dot{\omega} < \dot{x}$ q.e.d.

Proof of Equations (26) and (27): $\dot{\omega} = \dot{k} - \dot{h} = -(\delta + n) - \frac{\dot{h} - (m + n)h}{h} = -(\delta + n) - \frac{\kappa h^{1 - \alpha} - \epsilon}{h} + m + n = -\omega^\alpha + \chi \omega + m - \delta$.

$\dot{\chi} = \dot{c} - \dot{k} = (1 - \alpha)x^\alpha - m - \rho + \delta + n$.

Proof of Equation (35): Equation (19) $\Rightarrow \frac{dx}{dm} = \frac{1}{\alpha(1 - \alpha)x^{\alpha - 2}(1 + z)} \frac{dy}{df} = 0$. Equation (33) $\Rightarrow \frac{d\epsilon}{dm} = -\alpha(1 - \alpha)x^{\alpha - 2} \frac{dx}{dm} + \epsilon, \frac{d\epsilon}{df} = -\epsilon$.

Literatur


[Feichtinger/Hartl, 1986] Feichtinger, Gustav; Richard Hartl: Optimale Kontrolle ökonomischer Prozesse; de Gruyter.


[Steinmann/et al., 1996] Steinmann, Gunter; Alexia Prskawetz; Gustav Feichtinger: A Model on the Escape from the Malthusian Trap; Discussion Paper No. 25; Wirtschaftswissenschaftliche Diskussionsbeiträge; Martin-Luther-Universität Halle-Wittenberg