Appendix D: Transitional Dynamics

Equilibrium conditions that also hold out of the steady state:

From the low-skilled labor market equilibrium (24) and (7) with \( w_L \equiv 1 \), it follows

\[
c = \lambda (1-u)(1-s). \tag{D.1}
\]

Since \( u \) does not display transitional dynamics, this is also true for \( c \), hence from equation (4) it immediately follows that \( r_t = \rho \forall t \). From (10) – the vacancy creation condition – it follows

\[
q(\theta) = \frac{\alpha (1-\phi) c N}{(V_a - V_y) \lambda} = \frac{\alpha (1-\phi) (1-u)(1-s) N}{V_a - V_y}. \tag{D.2}
\]

From (14) and (21), it follows the relative profitability condition

\[
I = \frac{V_y}{B(1-\sigma_R)(V_a - V_o)}. \tag{D.3}
\]

Using (26), (23) to substitute for \( (1-n_A)(1-\phi) \), (24) to substitute for \( Z \), and using the property \( p(\theta)u = q(\theta)v \) of the matching function yields \( \dot{u} = 0 \). Hence, solving the RHS of (26) for \( u \) as a function of \( p, q \) and \( n_A \), using \( p(\theta) = q(\theta)\theta \), and dividing by \( u \) finally yields, after simplifying, the
following equilibrium $u$-$v$ relationship:

$$v = (1-u)(1-n_A)(1-\phi). \quad (D.4)$$

Alternatively, (D.4) could be derived from (23), (7) with $w_L \equiv 1$, and (24). Since $u$ does not display transitional dynamics, the ratio $v/(1-n_A)$ will not either.

The **high-skilled labor-market clearing condition** (25) can be rewritten – by using (21), (12), (11) and (14) – as follows:

$$w_H = \frac{n_s}{sN} \left( V_a - V_o + \frac{V_y}{1-\sigma_R} \right) \quad (D.5)$$

Finally, from the matching function $M(U,V) = AV^\eta U^{1-\eta}$, it follows $v/u = (A/q)^{1-\eta}$. Substituting into this equation for $v$ from (D.4) and for $q$ from (D.2), yields

$$u = \frac{(1-n_A)(1-\phi)}{A^{\frac{1}{1-\eta}} \left[ \frac{V_a - V_y}{\alpha (1-\phi)(1-s)N} \right]^{\frac{1}{1-\eta}} \cdot \left( \frac{1}{1-u} \right)^{\frac{1}{1-\eta}} + (1-n_A)(1-\phi)}. \quad (D.6)$$

Hence, $u$ can only be implicitly stated as a function of $n_A$, $V_a$, and $V_y$. However, we can use the implicit function theorem to derive the signs of $\partial u/\partial n_A$, $\partial u/\partial V_a$, and $\partial u/\partial V_y$, which we need for deriving the corresponding terms in the Jacobian matrix of the linearized dynamical system (D.20) below [still containing $u$ given implicitly by (D.6)]. Let’s define

$$F(u, n_A, V_a, V_y) \equiv \frac{1}{(1-n_A)(1-\phi)} \left[ \frac{A(V_a - V_y)}{\alpha (1-\phi)(1-s)N(1-u)} \right]^{\frac{1}{1-\eta}} - u = 0,$$

where we use in the following: $\left[ \frac{A(V_a - V_y)}{\alpha (1-\phi)(1-s)N(1-u)} \right]^{\frac{1}{1-\eta}} + 1 \equiv den = \frac{1}{u}$. Applying the implicit function theorem then yields
This negative relationship already follows from (D.4): holding $v$ constant, an increase in $n_A$ induces a decrease in $u$ (in and out of steady-state equilibrium). To further simplify the above expression, we note that from (D.6) it follows

\[ \frac{-1}{1-n_A} < 0 \]

Hence we get

\[ \frac{\hat{c}u}{\hat{c}n_A} = \frac{\frac{-1}{1-n_A}}{\frac{1}{(1-\eta)(1-u)} + \frac{(1-n_A)(1-\phi)}{u^2} \cdot \frac{u}{(1-n_A)(1-\phi)(1-u)}} = \frac{-\frac{1}{1-n_A}}{\frac{1}{1-\eta} + \frac{1}{u}} < 0. \]
Similarly:

\[
\frac{\partial u}{\partial V_a} = -\frac{\partial F/\partial V_a}{\partial F/\partial u} = -\frac{-[(1-n_A)(1-\phi)]^{-1} \cdot \frac{1}{1-\eta} \left[ \frac{A(V_a-V_y)}{\alpha(1-\phi)(1-s)N(1-u)} \right]^{1-\eta} \cdot \frac{A}{\alpha(1-\phi)(1-s)N(1-u)^2}}{\text{den}^2} - 1
\]

\[
= \cdots = \frac{-(1-u)}{(V_a-V_y)\cdot(1+\frac{1-\eta}{\alpha})} < 0.
\]

Finally, due to symmetry of the problem, we have

\[
\frac{\partial u}{\partial V_y} = \frac{1-u}{(V_a-V_y)\cdot(1+\frac{1-\eta}{\alpha})} > 0.
\]

**Dynamical System:**

The **industry flow equation** (5), \( \dot{n}_i = q(1-n_A) - In_A \), can be rewritten, by using (D.2) and (D.3), as follows:

\[
\dot{n}_i \left[ n_A, V_y, V_a, V_o, u(n_A, V_a, V_y) \right] = \frac{\alpha(1-\phi)(1-u)(1-s)N(1-n_A)}{V_a-V_y} - \frac{\rho V_a n_A}{B(1-\sigma_R)(V_a-V_o)}, \quad (D.7)
\]

with \( u \) given implicitly in (D.6).

Using (16), (9), \( r_t = \rho \forall t, (7) \) with \( w_L \equiv 1, (D.2), \) and (D.1), yields, after simplifying, the **young firms’ dynamic valuation equation**

\[
\dot{V}_y \left[ V_y, u(n_A, V_a, V_y) \right] = \rho V_y - \phi (1-u)(1-s)N(\lambda - 1 + \sigma_y), \quad (D.8)
\]

with \( u \) given implicitly in (D.6).

Using (18), (21), \( r_t = \rho \forall t, (6), (D.3), \) and (D.1) yields, after simplifying, the **adult firms’ dynamic valuation equation**
\[
V_a[V_y, V_o, u(n_d, V_y, V_o)] = \frac{2V_y}{B(1-\sigma_R)} + \rho V_o - (1-u)(1-s)N(\lambda - 1 + \sigma_a), \tag{D.9}
\]

with \(u\) given implicitly in (D.6).

Finally, using (20), \(r_t = \rho \forall t\), the definition of \(\pi_o\), (D.2), and (D.1), yields, after simplifying, the **old firms’ dynamic valuation equation**

\[
V_o[V_y, V_o, V_o, u(n_d, V_y, V_o)] = (1-\phi)(1-u)(1-s)N \left[ \frac{\alpha V_o}{V_o - V_y} - (\lambda - 1) \right] + \rho V_o, \tag{D.10}
\]

with \(u\) given implicitly in (D.6).

Hence, we have a system of four nonlinear first-order differential equations (D.7) – (D.10) in four endogenous variables \(n_d, V_y, V_o,\) and \(V_a,\) where the variables \(n_d, V_y\) and \(V_o\) also enter indirectly via (D.6).

**Steady-State Solution:**

Since we linearize our dynamical system around the steady-state equilibrium as can be seen in (D.20) below, it is useful to state the steady-state equilibrium solutions (indicated by “*” in the following) of our variables explicitly. In (B.1), we have already stated \(q\) as a function of exogenous parameters only:

\[
q^* = \frac{\alpha \rho}{\lambda - 1 + \frac{\sigma_a}{1-\phi} - \left[ \frac{2(\lambda - 1 + \sigma_y)}{\rho(1-\sigma_R)B} + \sigma_y \right]} \tag{D.11}
\]

By setting \(\dot{V}_y = 0\) in (D.8), we get the steady-state value for \(V_y\) as a function of the steady-state unemployment level only:

\[
V_y^* = \frac{(1-u^*)(1-s)N\phi(\lambda - 1 + \sigma_y)}{\rho}. \tag{D.12}
\]

Setting \(\dot{V}_a = 0\) in (D.9) and using (D.12) [or, alternatively, using (A.4), (7) with \(w_L \equiv 1,\) and the definition \(B \equiv \beta \delta / \gamma\)], yields the steady-state value for \(V_a\) as a function of the steady-state unemployment level only:

\[
V_a^* = \frac{(1-u^*)(1-s)N}{\rho} \left[ \lambda - 1 + \sigma_a - \frac{2\phi(\lambda - 1 + \sigma_y)}{B(1-\sigma_R)\rho} \right]. \tag{D.13}
\]
Using (A.5), (7) with $w_L \equiv 1$, (D.2), (D.12), and (D.13), yields, after simplifying, the steady-state value for $V_o$ as a function of the steady-state unemployment level only:

$$V_o^* = \frac{(1-u^*) (1-s) N (\lambda-1)(1-\phi)}{\rho \cdot \left( \frac{1}{1+\frac{\alpha(1-\phi)}{\lambda-1+\sigma_a-\phi (\lambda-1+\sigma_y)\cdot \left( \frac{2}{\lambda|1-\sigma_y|\rho} + 1 \right)} } \right)}.$$  \tag{D.14}

Next, we can use (D.3), (D.12), (D.13), and (D.14), to pin down the steady-state innovation rate as a function of exogenous parameters only (all unemployment terms cancel out):

$$I^* = \frac{\phi (\lambda-1+\sigma_y)}{B(1-\sigma_y) \cdot \left( \frac{2 \phi (\lambda-1+\sigma_y)}{\lambda-1+\sigma_a-\phi (\lambda-1+\sigma_y)\cdot \left( \frac{2}{\lambda|1-\sigma_y|\rho} + 1 \right)} \right)}.$$  \tag{D.15}

Setting $n_A = 0$ in (5), solving for $n_A$, allows to pin down the steady-state value of the share of type A firms:

$$n_A^* = \frac{1}{1+\frac{I^*}{q^*}}.$$  \tag{D.16}

with $I^*$ given in (D.15) and $q^*$ given in (D.11). (D.11) also allows to pin down the steady-state value of labor market tightness as

$$\theta^* \equiv \left( \frac{v}{u} \right)^* = \left( \frac{A}{q^*} \right)^{\frac{1}{\eta}}.$$  \tag{D.17}

and from this we get the steady-state job-finding rate as

$$p(\theta)^* = q(\theta)^* \theta^* = A^{\frac{1}{\eta}} q^{\frac{1}{\eta}}.$$  \tag{D.18}

The steady-state unemployment rate is then determined by the steady-state Beveridge curve equation (29) as follows:

$$u^* = \frac{1}{1+\frac{p^*}{1-\phi \left( \frac{1}{q^*} + \frac{1}{I^*} \right)}}.$$  \tag{D.19}
with $p^*$, $q^*$ and $I^*$ determined by (D.18), (D.11) and (D.15), respectively. Finally, using (D.19) in (D.12) – (D.14) yields the steady-state firm valuations $V_y^*$, $V_a^*$ and $V_o^*$, and using these together (D.16), (D.11) and (D.15) in (D.5) yields $w_{it}^*$.

**Linearized Dynamical System and Analysis of the Jacobian Matrix:**

Our dynamical system (D.7) – (D.10) linearized around the steady-state equilibrium looks as follows:

\[
\begin{bmatrix}
\dot{n}_A \\
\dot{V}_y \\
\dot{V}_a \\
\dot{V}_o
\end{bmatrix} =
\begin{bmatrix}
an_{11} & an_{12} & an_{13} & an_{14} \\
ban_{21} & an_{22} & an_{23} & an_{24} \\
ban_{31} & an_{32} & an_{33} & an_{34} \\
ban_{41} & an_{42} & an_{43} & an_{44}
\end{bmatrix}
\begin{bmatrix}
n_A - n_A^* \\
V_y - V_y^* \\
V_a - V_a^* \\
V_o - V_o^*
\end{bmatrix}
\]

(D.20)

with all entries $a_{ij}$ evaluated at the steady state (for our benchmark parameters with zero subsidy rates, where numerical analysis is needed to decide about the sign of any $a_{ij}$). We get:

\[
a_{11} \equiv \frac{\partial \dot{n}_A}{\partial n_A} = -\alpha \frac{(1-\phi)(1-s)N}{V_a^* - V_y^*} \left[1 - u^* + (1 - n^*_A) \cdot \frac{\partial u}{\partial n_A}\right] - \frac{V_y^*}{B(1-\sigma_R)(V_a^* - V_o^*)} \\
= \ldots = \frac{\alpha (1-\phi)(1-s)N}{V_a^* - V_y^*} \cdot \left[ 1 - \frac{1}{1 + \frac{1}{u^*}} \right] - \frac{V_y^*}{B(1-\sigma_R)(V_a^* - V_o^*)} < 0
\]

[Note that $V_a > V_y$ due to (D.2) for any positive $q$, and $V_a > V_o$ due to (D.3) for any positive $I$]; Mathematica gives us $a_{11} = -1.890062$.

\[
a_{12} \equiv \frac{\partial \dot{n}_A}{\partial V_y} = -\alpha \frac{(1-\phi)(1-s)N}{(1-n^*_A)\left(V_a^* - V_y^*\right)} \cdot \frac{\partial u}{\partial V_y} + \alpha (1-\phi)(1-u^*) \frac{\partial \dot{n}_A}{\partial n_A} - \frac{n_A}{B(1-\sigma_R)(V_a^* - V_o^*)} \\
= \ldots = \frac{\alpha (1-\phi)(1-s)N}{(1-n^*_A)\left(V_a^* - V_y^*\right)} \cdot \left[ \frac{1}{1 + \frac{1}{u^*}} - 1 \right] - \frac{n_A}{B(1-\sigma_R)(V_a^* - V_o^*)}
\]

Mathematica gives us $a_{12} = -0.102962$. 


\[ a_{13} \equiv \frac{\partial n_A}{\partial V_a} = \frac{-\alpha (1-\phi)(1-s)N(1-n_A^*)(V_a^*-V_y^*) \cdot \frac{\partial u}{\partial V_a} - \alpha (1-\phi)(1-u^*)(1-s)N(1-n_A^*)}{(V_a^*-V_y^*)^2} - \frac{V_y^*n_A^*B(1-\sigma_R)}{B(1-\sigma_R)(V_a^*-V_y^*)^2} > 0 \]

Mathematica gives us \( a_{13} = -0.009482 \).

\[ a_{14} \equiv \frac{\partial n_A}{\partial V_o} = -\frac{V_y^*n_A^*}{B(1-\sigma_R)(V_a^*-V_y^*)^2} < 0 \]

Mathematica gives us \( a_{14} = -0.058355 \).

\[ a_{21} \equiv \frac{\partial \dot{V}_y}{\partial n_A} = \phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{\partial u}{\partial n_A} = -\phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{\frac{1}{1-u^*}}{\frac{1}{1-\eta} + \frac{1}{u^*}} < 0 \]

Mathematica gives us \( a_{21} = -0.029335 \).

\[ a_{22} \equiv \frac{\partial \dot{V}_y}{\partial V_y} = \rho + \phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{\partial u}{\partial V_y} = \rho + \phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{1-u^*}{(V_a^*-V_y^*)(1+\frac{1}{\eta} + \frac{1}{u^*})} > 0 \]

Mathematica gives us \( a_{22} = 0.052979 \).

\[ a_{23} \equiv \frac{\partial \dot{V}_y}{\partial V_a} = \phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{\partial u}{\partial V_a} = -\phi(1-s)N(\lambda -1+\sigma_y) \cdot \frac{1-u^*}{(V_a^*-V_y^*)(1+\frac{1}{\eta} + \frac{1}{u^*})} < 0 \]

Mathematica gives us \( a_{23} = -0.002979 \).

\[ a_{24} \equiv \frac{\partial \dot{V}_y}{\partial V_o} = 0 \]

\[ a_{31} \equiv \frac{\partial \dot{V}_a}{\partial n_A} = (1-s)N(\lambda -1+\sigma_a) \cdot \frac{\partial u}{\partial n_A} = -(1-s)N(\lambda -1+\sigma_a) \cdot \frac{\frac{1}{1-u^*}}{\frac{1}{1-\eta} + \frac{1}{u}} < 0 \]

Mathematica gives us \( a_{31} = -0.332795 \).
\[
\begin{align*}
a_{32} &\equiv \frac{\partial \dot{V}_a}{\partial V_y} = \frac{2}{B(1-\sigma_R)} + (1-s) N (\lambda - 1 + \sigma_a) \cdot \frac{\partial \dot{u}}{\partial V_y} = \frac{2}{B(1-\sigma_R)} + (1-s) N (\lambda - 1 + \sigma_a) \cdot \frac{1-u^*}{(V_a^*-V_y^*)(1+\frac{1-u^*}{\sigma_a})} > 0 \\
\text{Mathematica gives us } a_{32} &= 0.441566. \\

a_{33} &\equiv \frac{\partial \dot{V}_a}{\partial V_a} = \rho + (1-s) N (\lambda - 1 + \sigma_a) \cdot \frac{\partial \dot{u}}{\partial V_a} = \rho - (1-s) N (\lambda - 1 + \sigma_a) \cdot \frac{1-u^*}{(V_a^*-V_y^*)(1+\frac{1-u^*}{\sigma_a})} \\
\text{Mathematica gives us } a_{33} &= 0.016206. \\

a_{34} &\equiv \frac{\partial \dot{V}_a}{\partial V_o} = 0 \\

a_{41} &\equiv \frac{\partial \dot{V}_a}{\partial n_A} = -(1-\phi)(1-s) N \left[ \frac{\alpha V_o^*}{V_a^*-V_y^*} - (\lambda - 1) \right] \cdot \frac{\partial \dot{u}}{\partial n_A} = (1-\phi)(1-s) N \left[ \frac{\alpha V_o^*}{V_a^*-V_y^*} - (\lambda - 1) \right] \cdot \frac{\lambda-1}{\frac{1-u^*}{\sigma_a} + \frac{1}{\sigma_a}} \\
\text{In the steady-state equilibrium, we can prove that the term } \frac{\alpha V_o^*}{V_a^*-V_y^*} - (\lambda - 1) \text{ is negative: first, we use (D.2) and solve for } V_o, \text{ which gives the condition} \\
V_o^* < \frac{(\lambda - 1)(1-\phi)(1-u^*)(1-s)N}{q(\theta)^*}. \\
\text{Plugging in the steady-state equation (A.5), using (24), and simplifying finally reduces the above condition to } \rho > 0, \text{ which is obviously true. It follows that sign}(a_{41}) < 0. \\
\text{Mathematica gives us } a_{41} &= -0.007700. \\

a_{42} &\equiv \frac{\partial \dot{V}_a}{\partial V_y} = \frac{-(1-\phi)(1-s) N \alpha V_o^*(V_a^*-V_y^*)}{(V_a^*-V_y^*)^2} \cdot \frac{\partial \dot{u}}{\partial V_y} + (1-\phi)(1-u^*)(1-s) N \alpha V_o^* \\
&= \cdots = \frac{(1-\phi)(1-s) N (1-u^*)}{V_a^*-V_y^*} \left[ \frac{\alpha V_o^* \left(1-\frac{1-u^*}{\sigma_a} \right) + \lambda-1}{V_a^*-V_y^*} + \frac{1-u^*}{\sigma_a} \right] > 0 \\
\text{Mathematica gives us } a_{42} &= 0.231038.
\end{align*}
\]
Mathematica gives us $a_{43} = -0.231038$. From (D.10) it follows immediately that $a_{43} = -a_{42}$.

Mathematica gives us $a_{44} = 1.970609$.

Now we want to make use of the fact that the determinant of the Jacobian matrix gives the product of its eigenvalues, and the trace of the Jacobian matrix gives the sum of its eigenvalues.

Developing the determinant of the Jacobian matrix with respect to the 4th column gives

$$
\text{Det}(J) = -a_{41} \cdot \text{Det} \begin{pmatrix}
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} \\
a_{41} & a_{42} & a_{43}
\end{pmatrix} + a_{44} \cdot \text{Det} \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
$$

$$
= -a_{41} \left( a_{22}a_{33}a_{44} + a_{23}a_{32}a_{44} + a_{24}a_{31}a_{43} + a_{32}a_{41}a_{43} - a_{41}a_{32}a_{23} - a_{42}a_{31}a_{23} - a_{43}a_{31}a_{22} - a_{43}a_{32}a_{21} \right) + a_{44} \left( a_{12}a_{23}a_{31} + a_{13}a_{22}a_{31} + a_{14}a_{21}a_{31} + a_{14}a_{21}a_{32} - a_{21}a_{32}a_{13} - a_{32}a_{21}a_{13} - a_{42}a_{31}a_{13} - a_{43}a_{31}a_{12} \right)
$$

By $\text{Det}(J) = x_1 x_2 x_3 x_4$, the sign of this determinant will be informative about the number of negative eigenvalues: if $\text{Det}(J) < 0$, it would follow that we have either one or three negative eigenvalues; if $\text{Det}(J) > 0$, it would follow that we have either zero, two or four negative eigenvalues. Using the numerical Mathematica results for our benchmark steady-state equilibrium as is required, we get $\text{Det}(J) = -0.00853$. Hence, we have either one or three eigenvalues with negative real parts.

By $Tr(J) = x_1 + x_2 + x_3 + x_4$, the sign of the trace of the Jacobian matrix will also be informative about the number of negative eigenvalues. Since

$$
\text{Tr}(J) = a_{11} + a_{22} + a_{33} + a_{44},
$$

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it follows that if $Tr(J) > 0$, not all four eigenvalues $x_{1,\ldots,4}$ could be negative, and if $Tr(J) < 0$, at least one eigenvalue must be negative. Using the numerical Mathematica results, we get $Tr(J) = 0.14973$. Hence, it follows that we have at most three eigenvalues with negative real parts, which is of course consistent with the above result that we must have either one or three negative eigenvalues.

Finally, we can use the Routh-Hurwitz theorem in the form stated by Gómez and Sequeira (2012, p.23)\textsuperscript{1}: The number of roots of the characteristic equation of $J$,

$$g(x) = x^4 - \delta_3 x^3 + \delta_2 x^2 - \delta_1 x + \delta_0,$$

with negative real parts is equal to the number of variations of sign in the scheme:

$$1, \quad \delta_3, \quad \Psi = \delta_2 - \frac{\delta_1}{\delta_3}, \quad \Pi = \delta_1 - \frac{\delta_2 \cdot \delta_3}{\Psi}, \quad \delta_0,$$

where the coefficients of the characteristic equation are as follows by Viète’s formulae:

$$\delta_0 = Det(J),$$

$$\delta_1 = a_{11} a_{22} a_{33} + a_{13} a_{44} a_{34} - (a_{11} + a_{22}) \cdot a_{34} a_{43},$$

$$\delta_2 = (a_{11} + a_{22}) a_{33} + a_{14} a_{22} - a_{34} a_{43},$$

and

$$\delta_3 = Tr(J).$$

We use this theorem to decide whether the number of eigenvalues with negative real parts is one or three. The numerical Mathematica results yield

$$\delta_1 = -0.001623$$ (note that $a_{34} = 0$ in our case)

$$\delta_2 = -0.129905$$

$$\Psi = -0.11907$$

$$\Pi = -0.01235$$

Therefore, the scheme that we need to look at from Routh-Hurwitz theorem is:

There is one variation of sign (from second to third element, the sign switches from positive to negative), hence we have **exactly one root (eigenvalue) with negative real part** in our characteristic equation $g(x)$. Hence we have **saddle-path stability**, with the **dimension of the stable arm** (equal to the number of negative eigenvalues) being equal to one. Since the three firm valuations can jump discretely to the values required to reach the saddle path (“jump variables”) but the share of type A industries $n_A$ cannot (“state variable”), the negative eigenvalue must belong to the variable $n_A$.\(^2\) This saddle path is unique since the dimension of the stable arm (number of eigenvalues with negative real part) equals the number of state variables, which here is one, namely $n_A$. To ensure that this saddle path is reached requires that we can freely choose as many initial values as we have unstable (i.e., positive) eigenvalues.\(^3\) This means that for appropriate starting values $V_{y,t=0}, V_{a,t=0}, V_{a',t=0}$, the system jumps onto the saddle path.

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\(^2\) Given that the unemployment rate $u$ does not display any transitional dynamics, it follows from (D.4) that the vacancy rate $v$ is a state variable displaying transitional dynamics that are exclusively driven by the dynamics in the term $1 - n_A$. Furthermore, it follows from (D.6) that on the transition path, the firm values $V_a$ and $V_y$ always adjust such that the dynamics in $1 - n_A$ are offset, so that $u$ does not display transitional dynamics.